The pendulum: from cooking spaghetti to a gravitational-wave detector

FRONTIERS

PCCP demonstrator

In the famous horror novel “The pit and the pendulum” by Edgar Allan Poe, a person is tortured with an oscillating instrument to the end of which a blade is attached.

Orienting and Asking Questions

Provide Contact with the content and/or provoke curiosity

Introduction

In this lesson, we will explore the pendulum, a very simple mechanical system, but a powerful tool to give deep insights in physics phenomena such as oscillations, gravitation, vibrations transmission, the concepts of speed, acceleration, energy, resonance.

We will first see how a pendulum can be used to measure time and this will allow to explore several properties of the pendulum.

At the end of the lessons we will also see how this device serves as the basis for gravitational-wave detectors such as LIGO and Virgo. These detectors succeeded in catching a gravitational wave for the first times between 2015-2017 (first LIGO and then LIGO and Virgo jointly), these little ripples in the fabric of space-time, predicted by Albert Einstein in 1916 and never observed before.

In LIGO and Virgo the pendula are used to reduce the transmission of ground vibrations to the instrument to a level compatible with the extreme sensitivity required by these detectors: a displacement of a billionth of a billionth of a meter over 4 km.

The measurement of time

A pendulum is a mass attached to a wire. Nothing more. A very simple system. In fact, there is an invisible ingredient: the gravitational field (the fact that the Earth exerts a force which makes objects fall down). When the pendulum is moved away from its equilibrium position, it starts to oscillate. Oscillations are regular movements. Counting oscillations means measuring time.

When an object oscillates, we can define two fundamental quantities: the oscillation period and frequency. The period of an oscillation is defined as the time that it takes to the pendulum to complete one full cycle (so to get back to the starting position). Usually, this quantity is measured in seconds. The frequency represents the number of oscillations that take place in the defined unit of time, let's say 1 second. The two quantities are then connected by the relation:

$$f = 1/T$$

Where T is the period. You can easily see that the frequency has the units of 1/s, also called 1 Hertz (1 Hz).

The measurement of time intervals has been an important issue for humankind since the beginning of its history.

What have been, in your opinion, the ways of measuring time in ancient times?

(and, by the way, why do we need to measure time?)

Teacher Guidelines:

- Hourglasses and sundials are very fascinating objects. If there is enough time, you can expand on this matter, maybe bringing a hourglass or showing images.

- you could also start a discussion about « what is time? ». The risk is a philosophical detour, but it can be interesting to hear opinions. Of course, here we talk about physical time (how the time enters in physical laws).

A funny experiment: cooking spaghetti with a pendulum

Suppose you don't have any clock and you want a mean to measure the time to cook your pasta. You can use a pendulum.

A pendulum is a way to measure time. You can say: "It takes 463 oscillations to cook spaghetti". Next time you cook pasta you can use the same number of oscillations.

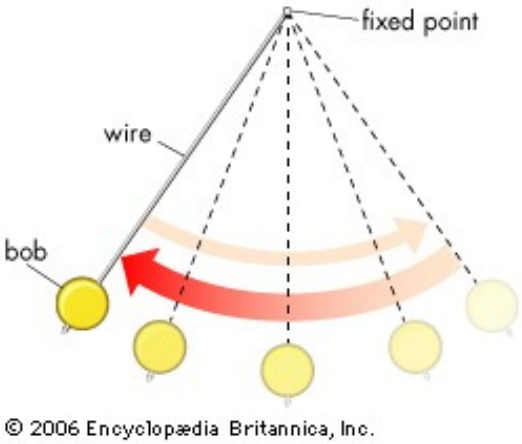
You can try to cook spaghetti with your pendulum. Taste the spaghetti while counting the oscillations of your pendulum. When the spaghetti are "al dente", stop. How many oscillations did it take?

In my own experiment, I have used a 0.37 cm pendulum. The spaghetti cooked in 463 oscillations and they were al dente.

Now you can compare your pendulum with a clock.

Measuring independently the frequency of oscillations, it was 0.75 Hz (hence, giving a period of 1.3 seconds). 463 oscillations correspond to 617 seconds = 10 minutes and 17 seconds.

According to the spaghetti producer (see pictures) the pasta will be al dente in 10 minutes (almost equal to the time measured with the pendulum).



Try to repeat the experiment. Do you obtain the same results?

Of course, other students will have different perceptions of “al dente” and they will have also built different pendula, so the numbers of oscillations for them will be different. But, for your own need, you don’t need another clock: the pendulum can perfectly be your clock.

On the other hand, if you want to share the measurement of time with others, you need two have equal pendula, which beat at the same time. How to make a shareable clock then? Which are the important properties of the pendulum?

The properties of a pendulum

Let’s forget the pasta for now and try to understand better which are the important properties that define the oscillations of a pendulum. Let’s ask ourselves:

- Do two pendula of the same mass have the same period?

- Do two pendula of the same length have the same period?

In the following, you will perform experiments and record your observations to answer to this question.

Design/Model

Take a thin wire and a ball. The wire should be thin, otherwise the elasticity of the wire will interfere with our measurement. The ball should not be too light, for the same reason. Not a ton, not a feather.

A mass and a wire are a quite simple system, with basically two parameters: the mass of the ball and the length of the wire (we have chosen a thin wire, which means, in the languages of physicists, a wire with zero diameter and with zero elasticity. If you take a thick or rigid wire (a rod), things are very different and that complicates the problem a lot).

Prepare a series of pendula: at least two with same mass but different wire lengths and at least two with the same wire length but different ball masses.

We are now ready to start our observations!

Perform investigations

Try to measure the period of the oscillation of all your pendula. You can use the chronometer of your mobile phone.

Now plot the oscillation period versus the masse of the pendulum and of the oscillation period versus the length of the wire. For each configuration, make several trials and average over the number of the trials.

Tip: when you plot the period vs length, it is better to plot the period squared versus length.

Analysis and Interpretation: Gather result from data

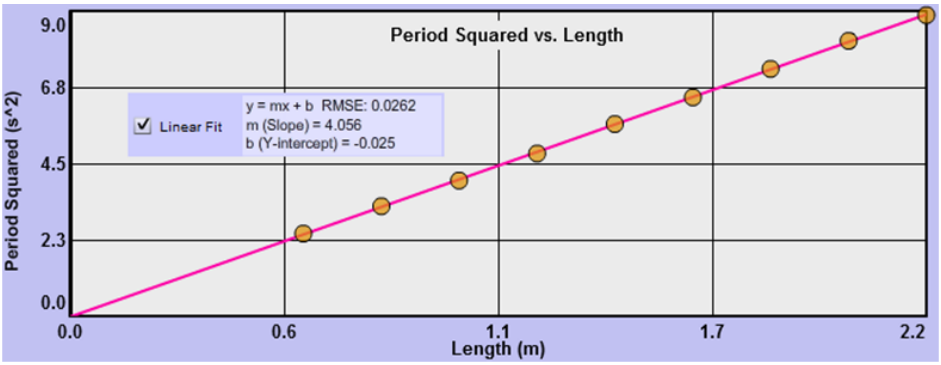
> Now that you have collected all your data and you have plotted them, what can you infer on the correlation between oscillation period and the two physical parameters of the pendulum, its mass and its length?

Are these results in agreement with your intuition or are they surprising to you?

After your investigations, what you should have found is that:

1. The period of the pendulum does not depend on the mass

2. The period squared depends linearly on the length of the wire



Teacher Guidelines

Depending on the level of your students, you can introduce here an explanation of gravitational field and force to help them understand the deep underlying reason for their observational results.

You now have a criterium to build a sharable clock: any two pendula of the same length will have the same period, independently from the mass of the ball.

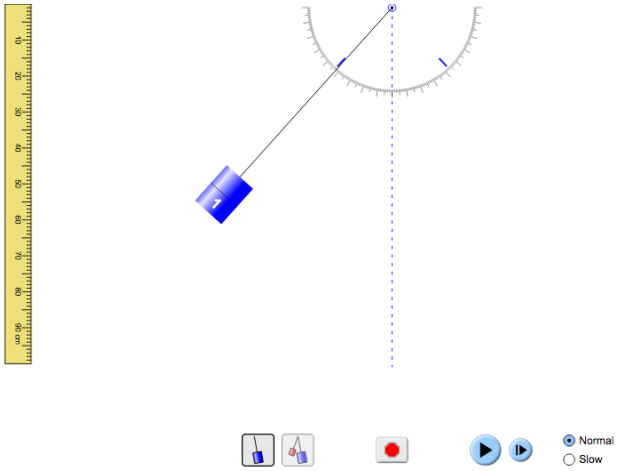
A virtual investigation

Reality is always more surprising (and complex) than the virtual world. But, sometimes, it is useful and instructive to simplify situations.

The PHET projects build simulation tools for teachers and students on various topics.

You can take a look to the pendulum simulation and play with the virtual pendulum.

The https://phet.colorado.edu/en/simulation/pendulum-lab



Conclude and communicate result/explanation

1. Write a 15 lines report on what you have done addressing the following points (context, methodology, results, discussions).
2. Record a 1 minute video or audio on what you have done addressing the following points (context, methodology, results, discussions). If you use a video, you can show your results (in the form of plots and schemes).

Evaluation/reflection

Historical note

Why is a meter a meter? In other words, why has the length unit been chosen to be exactly a meter and not half a meter or two meters?

The definition of the « meter » is the first attempt to build an objective measurement of length, based on physical quantities rather than human quantities (before that, people were using more “anthropocentric” units such as the inch or the foot).

One meter corresponds to the length of a pendulum whose beat is 1 second (period is 2 seconds). Is this in agreement with your intuition?

The pendulum and gravitational-wave detectors

To have a wider understanding of gravitational waves and gravitational-wave detectors, please, you can refers to the set of demonstrators: ““Finding black-holes in a chirp”: how to understand the first gravitational-wave detection”, “Discovering and building a Michelson interferometer”, “Gravitational Wave noise hunting”, “Virgo Virtual Visit”, and finally “Control (class)room”.

Earth vibrates, even if we don't feel it. This is due to the interaction between the solid crust, the atmosphere and the oceans. The vibration of the Earth can be measured with seismometers and gravimeters and it is about 1 micron every 10 seconds, in a standard environment. This seems small, but for Virgo and LIGO is a huge displacement, compared with the gravitational-wave amplitudes.

In order to alleviate the effect of Earth's vibrations, Virgo and LIGO use pendula.

Other than measuring time, a pendulum is also basically a filter: when the suspension point is shaken, it filters out the high frequency vibrations. Only the low frequency vibrations (long periods) are transmitted.

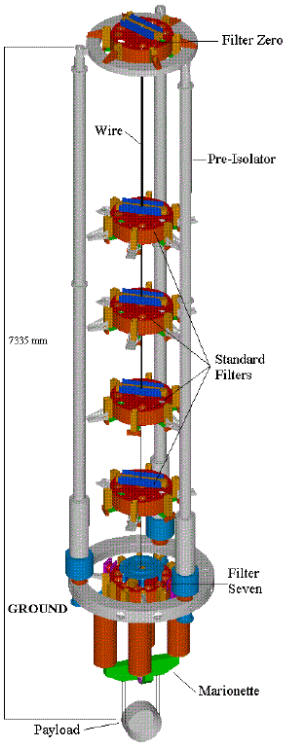
If you shake the suspension point of a pendulum very slowly, the mass will follow you. When you shake the suspension point with a very fast movement (periods smaller than the period of the pendulum), the ball vibrates less than the pendulum.

Try with your pendula and describe your observations.

You can watch the Nobel prize Rei Weiss doing the experiment during the press conference about the first gravitational-wave detection, in February 2016 (at min 28)

https://www.youtube.com/watch?v=aEPIwEJmZyE

In gravitational-wave detectors, much more sophisticated pendula are used.



pendula used in the gravitational-wave detector Virgo.

In order to improve the attenuation of the pendulum, you can use a multi-pendulum chain: a pendulum suspended to a pendulum, suspended to a pendulum, 7 times (or more).

When the Earth vibrates quickly, the vibration is transmitted through the pendulum chain to the mirror, but the mirror does not move. It's a free mass, as if it is was in the interplanetary space free of gravity!

With a pendulum we have (at least for quick vibrations) disconnected the mirror from the ground.

Conclusions

We have seen how a simple system as a pendulum (a ball attached to a wire) is able to measure time (to be a clock) and also to attenuate fast displacement of the ground, in order to allow the measurements of gravitational-waves.