Relativistic Muons and Time Dilation

E. Chaniotakis, RnD Department, Ellinogermaniki Agogi

Prior knowledge: Particle decay, particle lifetime distribution, properties of muons, basic kinematic relationships. Basic algebraic manipulations. The “eV” unit of energy and its multiples.

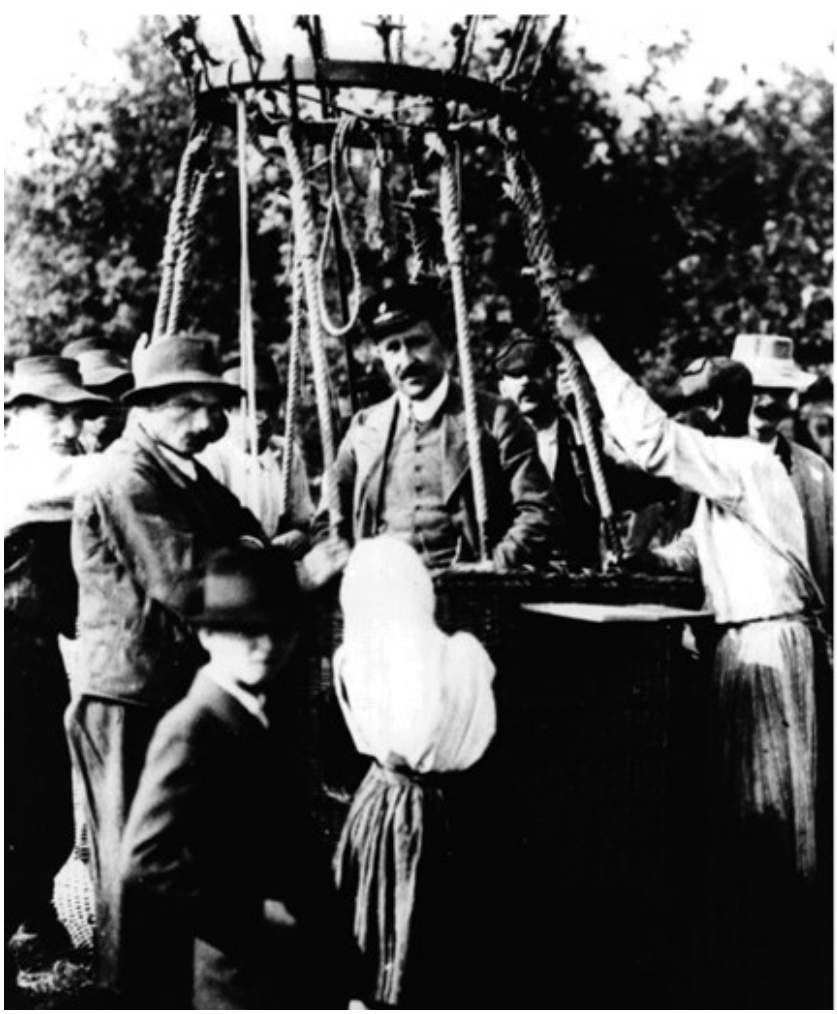
Educational objectives:
- Students learn about the relativistic time dilation using muon decay in flight.
- Students understand why we observe atmospheric muons at sea level, contrary to the predictions of Classical Physics.

Orienting & Asking Questions

Orienting: Provide Contact with the content and/or provoke curiosity

During the first years of the 20 th Century, dedicated Physicists flying balloons with

electrometers and radiation detectors observed a thin rain of particles coming from the cosmos. Nucleons and atomic nuclei coming from outer space, constantly collide with the molecules of the atmosphere resulting in the production of new particles which can be measured by airborne and ground detectors.



Picture 1.1: Victor Hess before a balloon flight in 1912 in Austria

The origin of these cosmic rays still remains a mystery to science, while their energies may exceed these of the most powerful particle beams at CERN by many orders of magnitude!

Watch this video to learn more about cosmic rays!
https://www.youtube.com/watch?v=ao-CzSrtqMU

Teacher Guidelines
You can check the history of cosmic ray discoveries and stimulate further discussion with the students at this link:
http://timeline.web.cern.ch/timelines/Cosmic-rays?page=1

Enter the Muon!

It was 1936 when Carl Anderson and Seth Neddermeyer of Caltech discovered a new particle: the Muon. Without any theory predicting its existence, this new particle was met with so much surprise, that the Nobel Prize Winner Isidor Rabi quoted: "Who ordered that?"

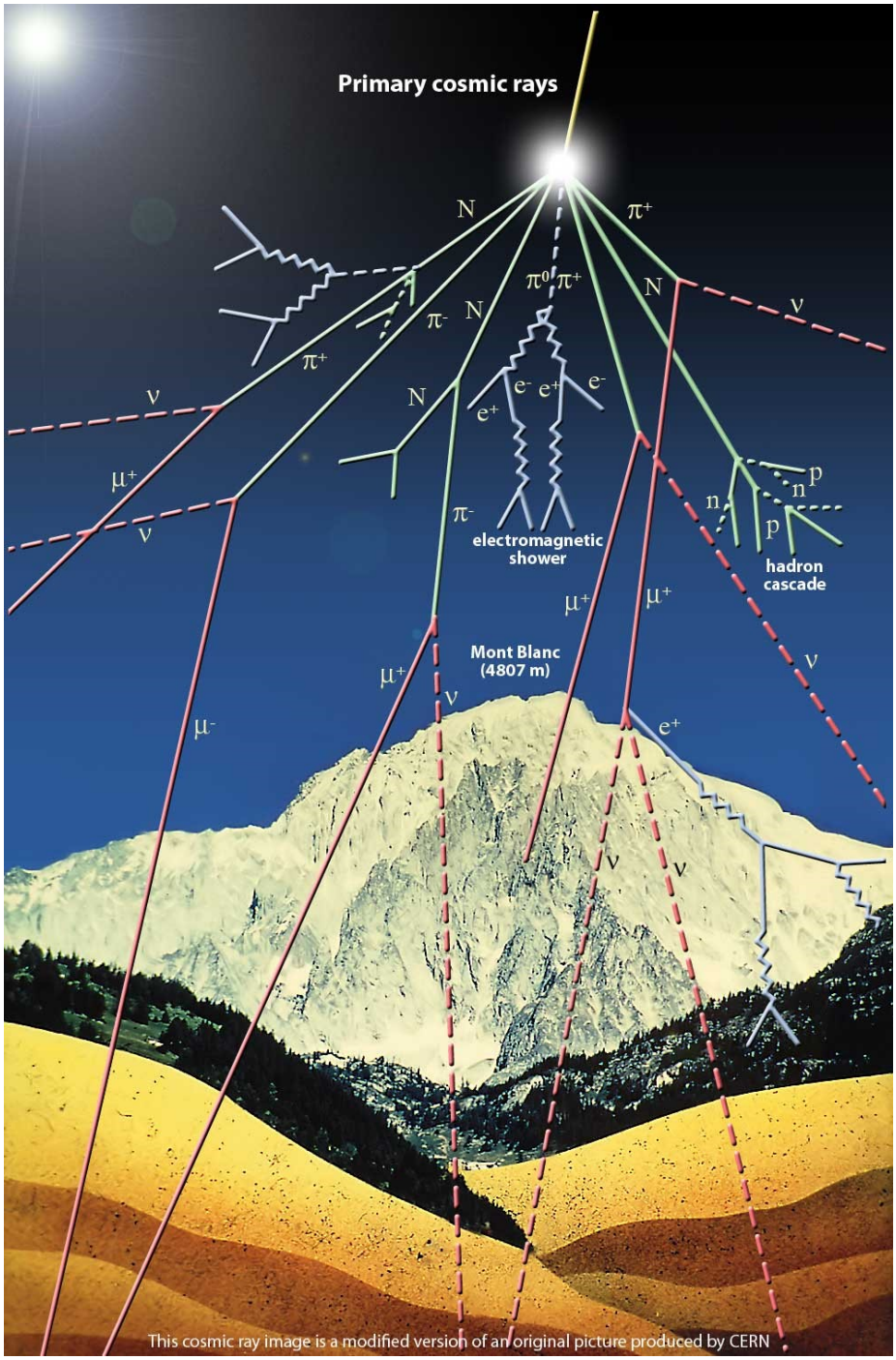
The muon is a "heavier cousin" of the electron with mass equal to 105,66 MeV/c^2 (200 times heavier than the electron) and negative charge equal to that of the electron (q_e=-1,6*10^{-19} C). Muon has an antiparticle: the anti-muon which has the same mass as the muon and positive charge of equal absolute value.

Muons are produced in interactions of cosmic rays with the molecules of the atmosphere at altitudes around 15km and reach the surface of the earth travelling with speed close to the speed of light and with kinetic energies distributed around 4 GeV day and night. The muon rate at sea level is one muon per minute per cm^2.

Besides atmospheric muons, accelerator experiments which collide elementary particles at high energies also result in copious production of muons in the laboratory.

Question

Comparison of a "muon panel" vs a "solar panel".
Approximately 1 muon with average energy equal to 4 GeV reaches earth at sea level per cm^2 per minute. Assume that scientists were able to create a 100% efficient "Muon Panel" which is able to stop muons and convert all their energy into electric energy.
If the panel has size: 2m * 1 m and operates for 24 hrs , find how much energy is deposited in it. Convert the energy in Joules.
Compare the power output of the panel to the output of a typical solar panel which is equal to ~3kW.
Discuss: What do you think? Can we harness the energy of cosmic rays for the benefit of society?



Picture 1.2: Development of a cosmic ray shower. A primary cosmic particle interacts with atomic nuclei of the atmosphere's constituents, producing a "shower" of new particles which further interact and decay. Out of these particles, muons are those that arrive at sea level with the highest abundance.

Muon range in matter

Teacher guideline

Due to their high mass, muons lose energy mainly by ionization of the medium they traverse.

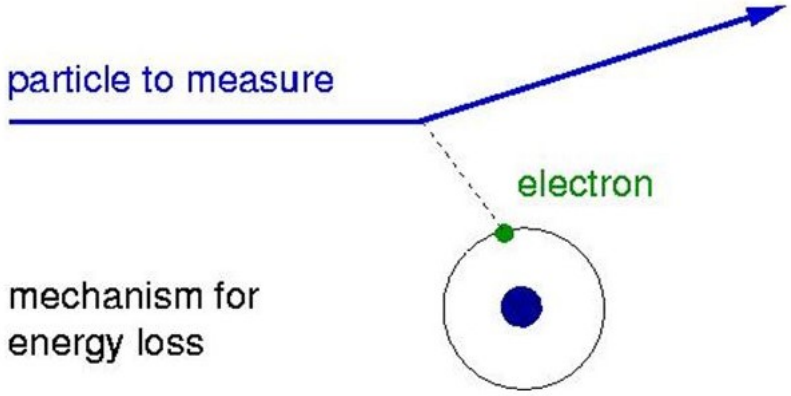
The energy loss for muons in the energy range of interest (a few GeV) for our studies is of the order: $\langle \Delta E/\Delta x \rangle = 2\ MeV/g*cm^{-2}$.

The g/cm^2 unit is called "column density". Displaying the energy loss with respect to this unit, makes our calculation independent of the medium we observe.
In order to find how much energy muons lose per cm of their path in any medium, we multiply $\langle \Delta E/\Delta x \rangle$ with the density of the medium in question.

As an example: For water , the density is equal to $\rho_{water} = 1\ g/cm^3$.
Muon energy loss in water = $2\ MeV/gcm^{-2}*1\ g/cm^3 = 2\ MeV/cm$

Try this for muons propagating in the air: $\rho_{air} \sim 1.225*10^{-3}\ g/cm^3$.
If the muons are produced at 10km altitude, how much energy have they lost when they reach sea level?
If the muons arrive at the earth with energies equal to 4 GeV, how much energy did they have when they were produced?
How far will a 4 GeV muon travel in the air before losing all its energy? What about water?

A heavy charged particle such as a muon, with energy within the energy range of interest for the purpose of our discussion, travelling through a medium will lose energy due to its electromagnetic interaction, mainly with the electrons of the medium's atoms. This interaction will result in ionizations of the atoms and formation of electron-ion pairs, or in excitations of atomic electrons which will move to a higher energy orbit and then de-excite by emitting low energy photons. Due to the muon's large mass and small charge, the energy loss per unit length in a medium is very small. As a result, the muon has very long range in matter.
Imagine: In order to stop a muon produced in cosmic ray interactions it will take about 30 km of air!



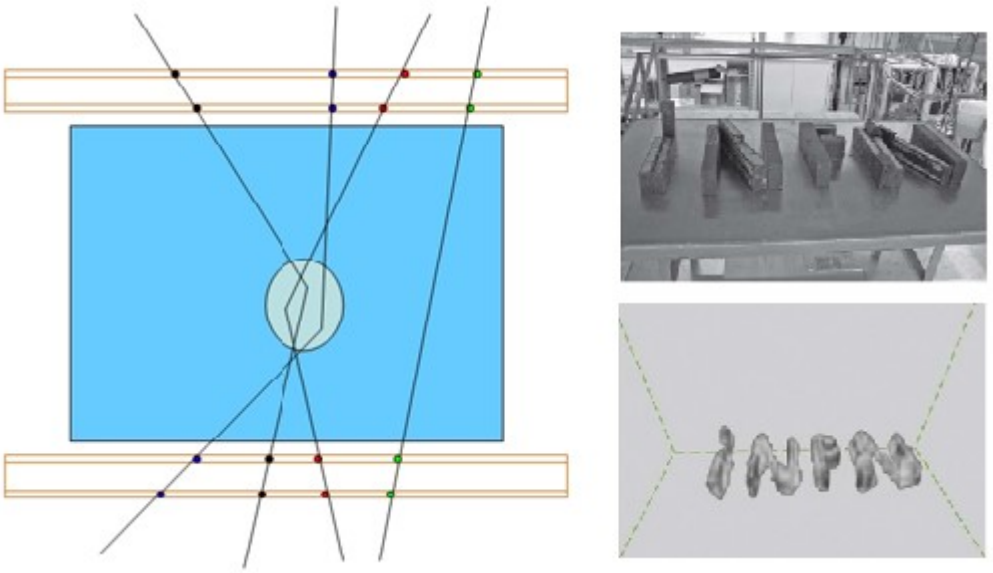
Picture 1.3: Energy loss of a muon through ionization.

Due to its high mass, a muon travelling through a medium will not undergo significant multiple scattering, unlike the electron.
The combination of low ionization yield and small multiple scattering results in muon trajectories being long and straight. This can be dramatically visualized in a cloud chamber capturing high energy muons coming from the atmosphere:

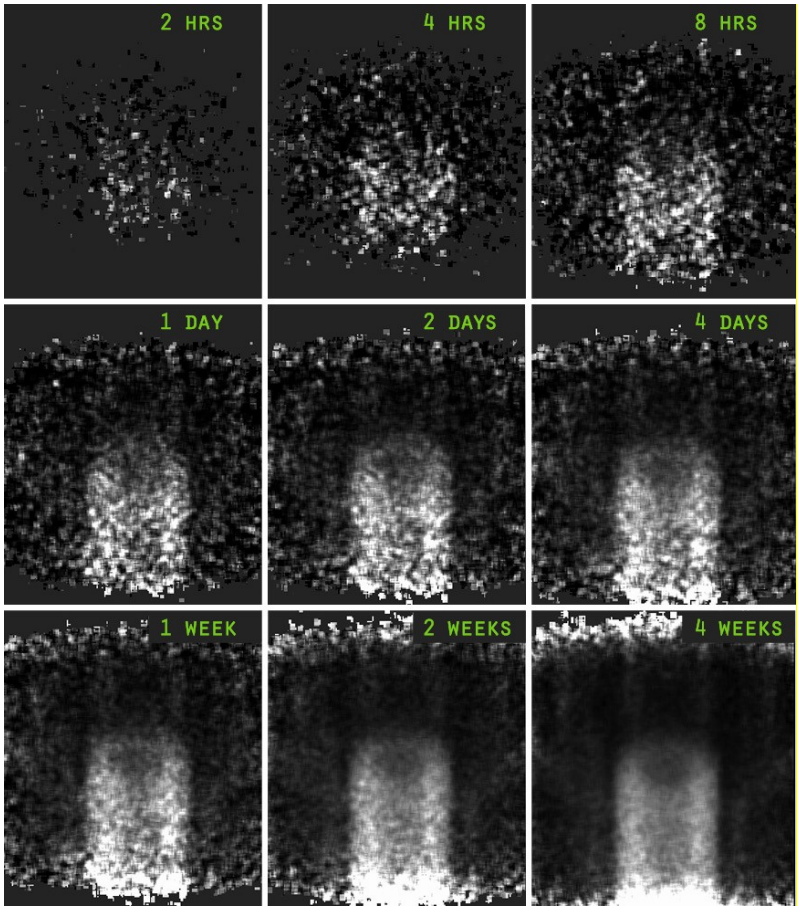
Picture 1.4: A muon track captured by a cloud chamber

“Atmospheric” Muons’ long range in matter is considered a powerful experimental signature by which scientists can detect muons using dedicated setups.
This characteristic is also employed in many aspects of modern research. Such an example is muon tomography: a technique very similar to the X-Ray tomography which uses muons produced in the atmosphere to “photograph” the innards of volcanoes, destroyed power plants, hidden explosives in ordinary containers or even search for hidden chambers in pyramids!

Teacher Guideline
This is done by placing muon detectors at the two sides of the object that scientists want to observe and measuring the distributions of muons as they enter and as they exit the object. The scattering of muons within the object can reveal the amount of matter that the muon met during its path and thus the existence of heavy elements or hidden cavities.



Picture 1.5: Left: Muons (straight lines) travelling downwards enter the first detector (up brown block), then the sample (blue block) and then the second detector (down brown block). Heavier elements create larger scattering. As a result, by comparing the incoming with the outgoing muon angular distribution, scientists can see the innards of an object.
Right: The word INFN (The name of the Italian Institute for Nuclear Physics) made by lead blocks. Up: The lead blocks. Down: The word reconstructed with muon tomography



Picture 1.6: Muon tomography of a nuclear reactor in Kawasaki, Japan

Define Goals and/or questions from current knowledge

Muon Decay

We have learnt that muons are heavy cousins of electrons which are produced during the interactions of cosmic rays with atomic nuclei of the atmosphere's constituents. They have a long range in matter and arrive at sea level with a flux of 1 muon per minute per cm^2.
The flux increases in higher altitudes as you can see from this plot:

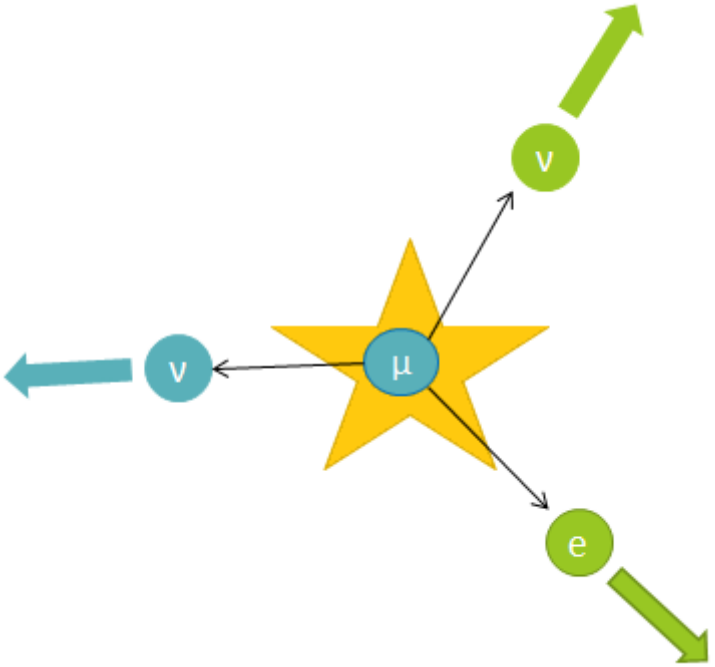
Discuss the following:
If muons are so abundant, why does ordinary matter not consist of them but only of protons, neutrons and electrons?

→ Because muons decay!!

You can imagine that muon decay is like popcorn popping.
https://www.youtube.com/watch?v=LG830o8AMudI

Like minuscule bombs, the muons have a finite mean lifetime of 2,2 billionths of a second – or μs. Some decay quicker, some slower. By stopping a big quantity of muons and observing their decay rate, the mean will be of the order of 2.2 μs.
Imagine an internal "clock" ticking inside them - and once this is over, they decay by transforming to three new particles which were not constituents of the parent muon: an electron and two other neutral particles called neutrinos.
As momentum and energy are always conserved, the daughter

particles share the momentum and the energy of their mother particle.



Picture 1.5 : Illustration of a muon decaying to an electron (e-) and two particles called neutrinos. Observe that one neutrino has the same color with the electron and the other has the same color with the muon. These two neutrinos are.. a little bit different among them.

Question:

If the parent muon is at rest, how much energy and momentum will the daughter particles share?

Answer:

According to Einstein's Special theory of relativity, the energy of a particle at rest is equal to its mass, m, times the speed of light squared, c^2: $E = mc^2$.

We name the particles: muon (μ), electron (e), neutrino 1 (ν), neutrino 2 (ν').

By applying energy conservation:

$E_\nu + E_{\nu'} + E_e = m_\mu c^2$

Since the parent muon is at rest, its momentum will be equal to zero. Therefore by applying momentum conservation:

$\mathbf{p}_\nu + \mathbf{p}_{\nu'} + \mathbf{p}_e = \mathbf{0}$

Hypothesis Generation and Design

Generation of Hypotheses or Preliminary Explanations

Are you familiar with the concept of radioactive decay?
Have a look at this video:

https://www.youtube.com/watch?v=kcVThHXqveI

The law of radioactive decay can be synopsized as:
"Radioactive decay is a random process at the level of single particles, in that, according to quantum theory, it is impossible to predict when a particular particle will decay, regardless of how long it has existed. However, for a collection of particles, the collection's expected decay rate is characterized in terms of their measured decay constants or half-lives."

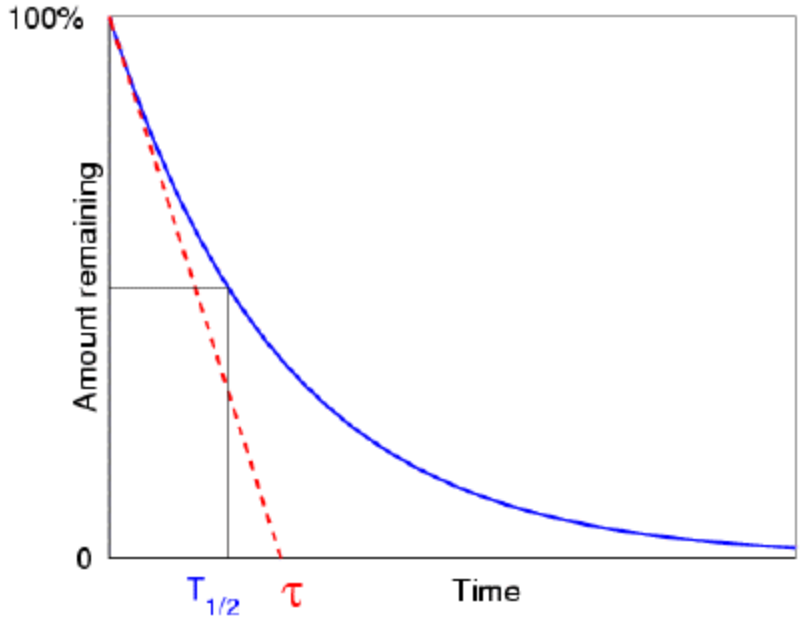
Teacher guideline
The teacher can refer to this link which beautifully explains radioactive decay
http://math.ucr.edu/home/baez/physics/ParticleAndNuclear/HalfLife/halfLife.html
An initial sample of No particles undergoes radioactive decay. After time equal to t, the

number of particles remaining in the sample without having decayed is : N.
The relationship between N, No and t is given by the exponential law of radioactive decay:

$$N(t) = N_0 e^{-\lambda t}$$

λ is the decay constant which determines the decay rate of the sample in question.

The law of radioactive decay can be depicted in the following graph:



Picture 2.1 : Graph representing the law of radioactive decay.

$$T_{1/2} = \frac{\ln 2}{\lambda} \approx \frac{0.693}{\lambda} \approx 0.693\tau$$

Radioactive half-life

Radioactive decay constant

Mean lifetime

Question:
Find the units of the decay constant λ. Describe what happens when λ is a large or a small number.

Answer
The decay constant is measured in s^{-1}. If λ is small, the sample will take a long time to decay, whereas if λ is a large number, the sample will deplete quickly.

Question:
What is the half-life of a radioactive sample?

Answer
Half life, $T_{1/2}$ equals the time needed for half the nuclei of the sample to decay.

Question:
What is the mean lifetime of a radioactive sample?

Answer:
Mean lifetime, τ, equals the time needed for the total number of nuclei in the sample to drop to 1/e of the initial quantity. It is equal to the average time under which we expect a particle of the sample to decay.

Let's discuss muon decay a bit further:
Muon decay is a random process obeying the law of radioactive decay.
The most striking characteristic of radioactive decay is its inherent randomness, which is pretty astounding by itself, and is a core feature of quantum mechanical systems. Therefore, if you "catch" two muons and measure how long it will take them to decay you may find totally different results. If you take data on the decay of many muons and average them over many single decay events, you will be able to find the characteristic time – or lifetime of the muon.

This is the main difference between a clock which needs two readings to define a time interval and a "radioactive clock" which needs a collection of events from which the characteristic time will be calculated.

Physics predicts that if the "mechanism" behind the decay is strong then the decay will probably happen fast: Our collection of muons will deplete quickly.
If the mechanism is weak, our collection of muons will gradually deplete over a much longer timescale.

Teacher Guideline
Muon decay is a manifestation of the "weak interaction". This implies that the decay rate of muons is rare, therefore muons tend to live rather long.
How long is long?
Processes involving the nuclear force take times as short as 10^{-23} sec, electromagnetic interactions take times of the order of 10^{-16} sec while weak interactions take as long as 10^{-10} sec.
Therefore, muon decay is a very slow process compared to other processes of the microcosm.

Question:
Does mean life τ =2.2μs imply that all muons will decay in 2.2μs after their creation?

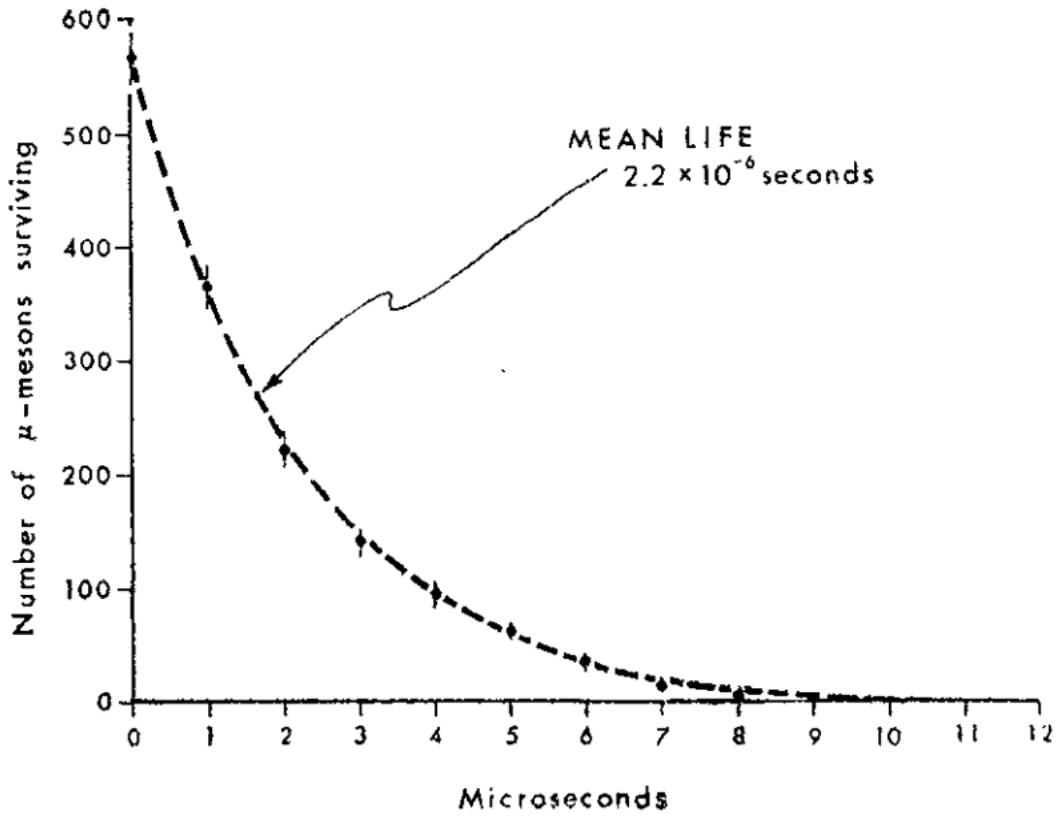
Answer:
No. The probabilistic nature of radioactive decay implies that the decay of a particle at the next moment doesn't depend on how long the particle has lived so far. A particle may decay spontaneously or after a long time. If, however, the particle has a mean lifetime of 2.2μs it will have probably decayed by then.
By transforming our question to : what do I expect to see when I have a large collection of muons and observe them, the answer is that by 2.2μs I will expect most of them to have decayed (1/e of them will remain).

Design Model

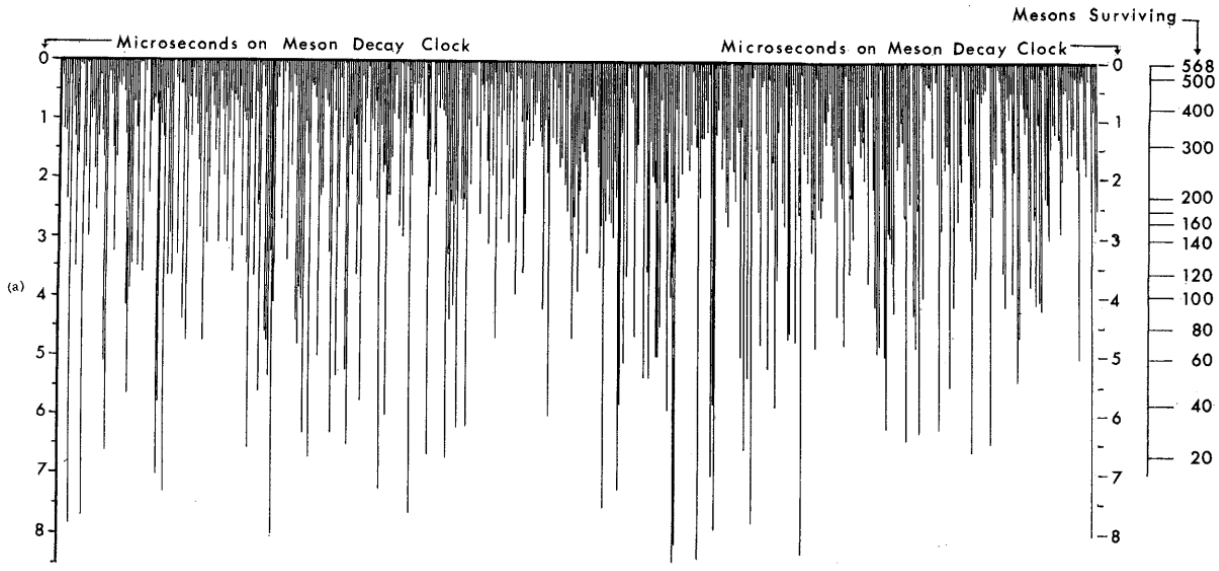
Let us observe the results of muon decay at rest obtained by a group of experimenters in the early 60's. The experimenters placed their muon detector at the top of Mt. Washington at an altitude of 1910 m and stopped them. The muon rate per hour was determined to be equal to 568 muons.
After stopping the muons, the researchers observed the time interval between their capture and their decay. From this, they could observe the time distribution of the muon decay process.

Teacher guidelines
The experiment, called "Time dilation, an experiment with mu-mesons" was an educational experiment conducted by D. Frisch and H. Smith of MIT in the early 60s'. It repeated the classic experiment of Rossi in 1941 with which he proved the relativistic time dilation. You can see the full documentary of this experiment here: https://www.youtube.com/watch?v=tbsdrHlLfVQ

In pictures 2.3 and 2.4 you will see the time distribution of muon decays



Picture 2.3: The exponential distribution of muon decays. We observe that more than 50% of the stopped particles will have decayed by a time equal to 2.2μs whereas some muons will take longer than 6μs to decay.



Picture 2.4: Distribution of decay times for muons. The vertical axis measures the decay interval of stopped muons in microseconds whereas the ruler on the right calculates how many muons have survived for the corresponding time interval.

(Figures Adapted from "*Measurement of the Relativistic Time Dilation Using μ-Mesons" David H. Frisch and James H. Smith. Citation: American Journal of Physics 31, 342 (1963); doi: 10.1119/1.1969508*)

Now let us make a calculation using picture 2.4.

Assume that the muons observed in Mt. Washington travel 1910 m vertically to the level of the sea. What do you expect to happen to them?

Teacher Guideline:
It is expected that most of them will decay in flight.

How long should a muon live in order to be able to travel the 1910m distance? Assume that its velocity is equal to 0.992c (c = 3.10^8m/s)

Teacher guideline:

$$t = \frac{x}{c} = \frac{1910\ m}{0.992 * 3 * 10^8\ {}^{m}/_{s}} = 6.42\ \mu s$$

The above results imply that only the "long lived" muons with lifetime equal to the number you calculated will manage to reach sea level.
Can you estimate the number of muons you expect to reach the sea level in an hour if we assume that initially: 568 muons were created at 1910 m altitude?

Teacher guideline:
Use a ruler and place its bottom side on picture 2.3 at a time equal to 6.36 μs. Use it to measure how many "lines" representing muons which have survived at least 6.36 μs. You will find that the number is roughly equal to 27 muons.

As a result: For every 568 muons collected at a 1910 m altitude with average velocity equal to 0.992c, 27 were expected to be observed at sea level.
After that, the researchers took their experimental setup at sea level and performed the one hour long measurement of muon decay at rest again.
What would you expect them to observe?

In the following activity you will emulate this experiment and figure out the answer yourself.

Planning and Investigation

Plan Investigation

Before the beginning of the activity, we need to address the topic of atmospheric muon flux. Muon flux at sea level is equal to 1 muon per minute per cm². With increasing altitude, the muon flux increases exponentially.
The number of muons observed in our detector largely depends on the run time and the detector area, efficiency and geometry.

Teacher guideline
The flux is higher at higher altitudes because downgoing muons decay or get absorbed by the atmosphere.You can see this resource for an experiment carried out in order to measure the muon flux vs a series of parameters among which the altitude: http://www2.lbl.gov/abc/cosmic/SKliewer/Index.htm

Question:
How many muons will we count with a 10 cm² area detector per hour at sea level? How does the muon count rate depend on the area of the detector and the detection time?

Answer:
Since the flux is 1 muon/minute. cm² , we expect to count at most:

$$Nr\ of\ muons = \frac{1\ muon}{minute\ .cm^2}\ 10cm^2 60min = 600\ muons$$

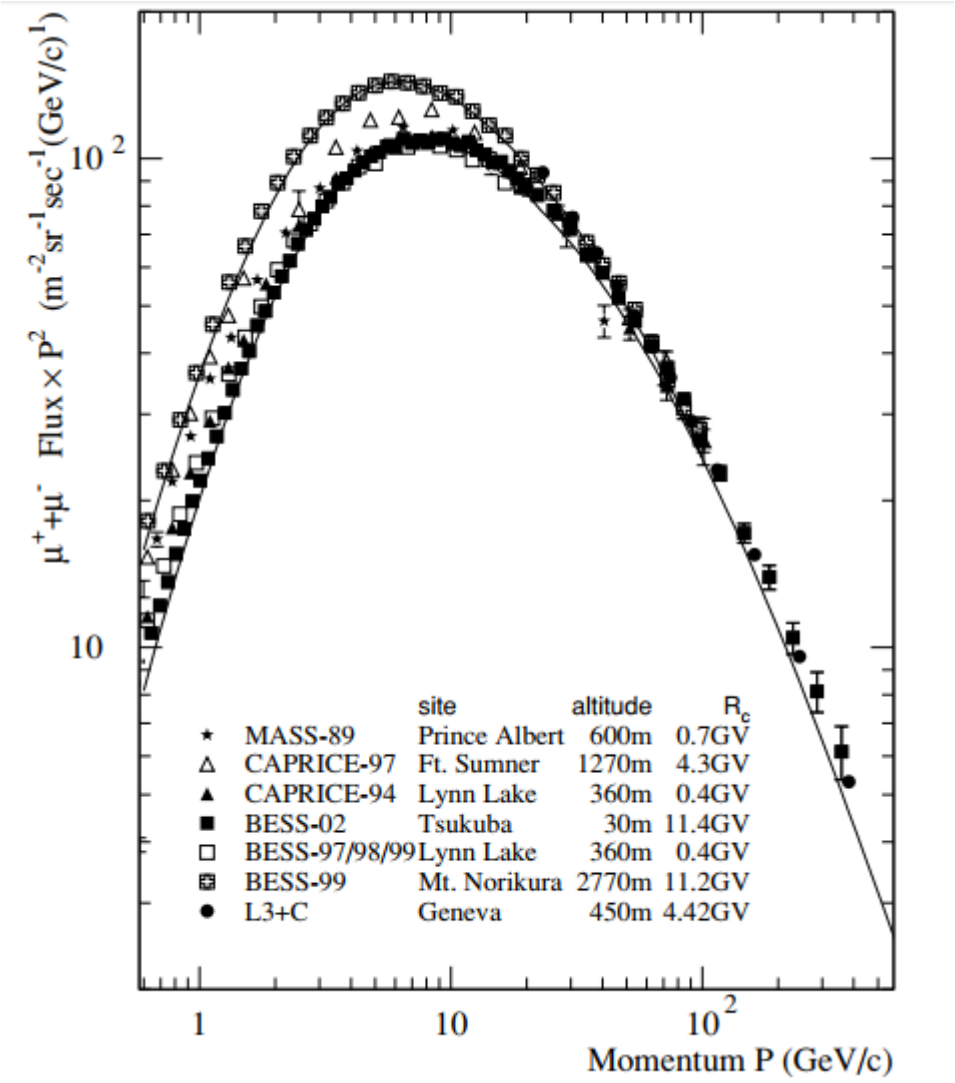
The muons will be actually less, because the detection efficiency will also play a significant role (in our assumptions we consider that we collect all muons that arrive at our setup, which is never true in a real experiment). Therefore our real result will be the number we observed multiplied by ε , the detection efficiency.
Furthermore, the detector geometry is very important as it constrains the detector's field of view. So far, we assume that we detect all downgoing muons coming from a 2π solid angle. However, this is not the case in real experimental setups.
Therefore a Geometry factor, G≤ 1, depending on the detector layout will also be multiplied with our result.

Therefore:

$$Nr\ of\ muons = Geometry\ Factor * Efficiency * Flux\ \left(\frac{muons}{minute\ .cm^2}\right) * Detector\ Area(\ cm^2) * Time(min)$$

Another factor that should also be taken into account is the energy distribution of muons. As the muon flux
muon count rate we will observe.

Frisch and Smith used their setup to measure muons in the velocity range of 0.99c to 0.994 c which corresponds to an energy range of 0.74 to 0.96 GeV/c.



(Figure adapted from: T. Samuki: Atmospheric Muon Fluxes at Various Locations)

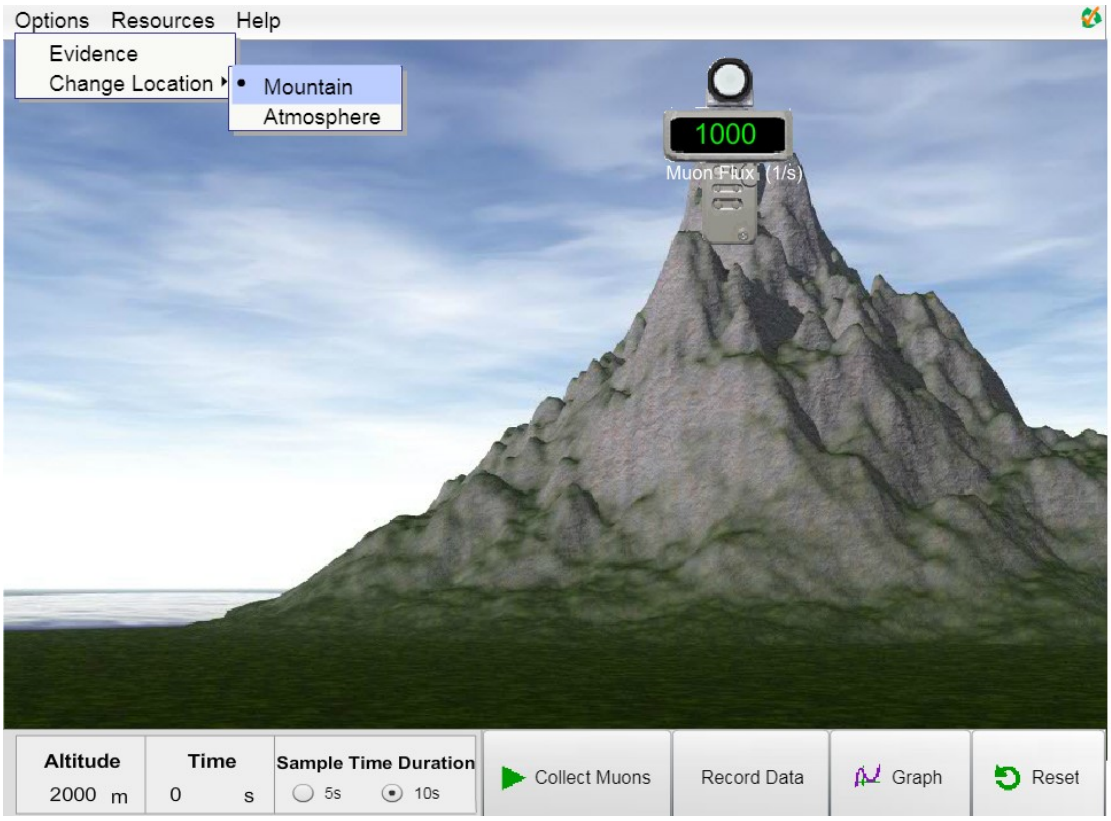
The Frisch –Smith experiment, an educational adaptation of Rossi's experiment in 1941 measured muon decay at rest in different altitudes. Such an
In this activity we will emulate this experiment using the KCVS muon decay simulation.

http://www.kcvs.ca/site/projects/physics_files/specialRelativity/muonDecay/muonDecay.swf

Take some time to get to know the various controllers of the simulation.

Perform Investigation

- Choose: Options\ Change Location \ Mountain



Picture 3.1: The "Mountain" environment of the muon decay simulation.

- Choose: Sample time duration = 10 s.

- Starting from an altitude of 2000m, press: "Collect Muons"

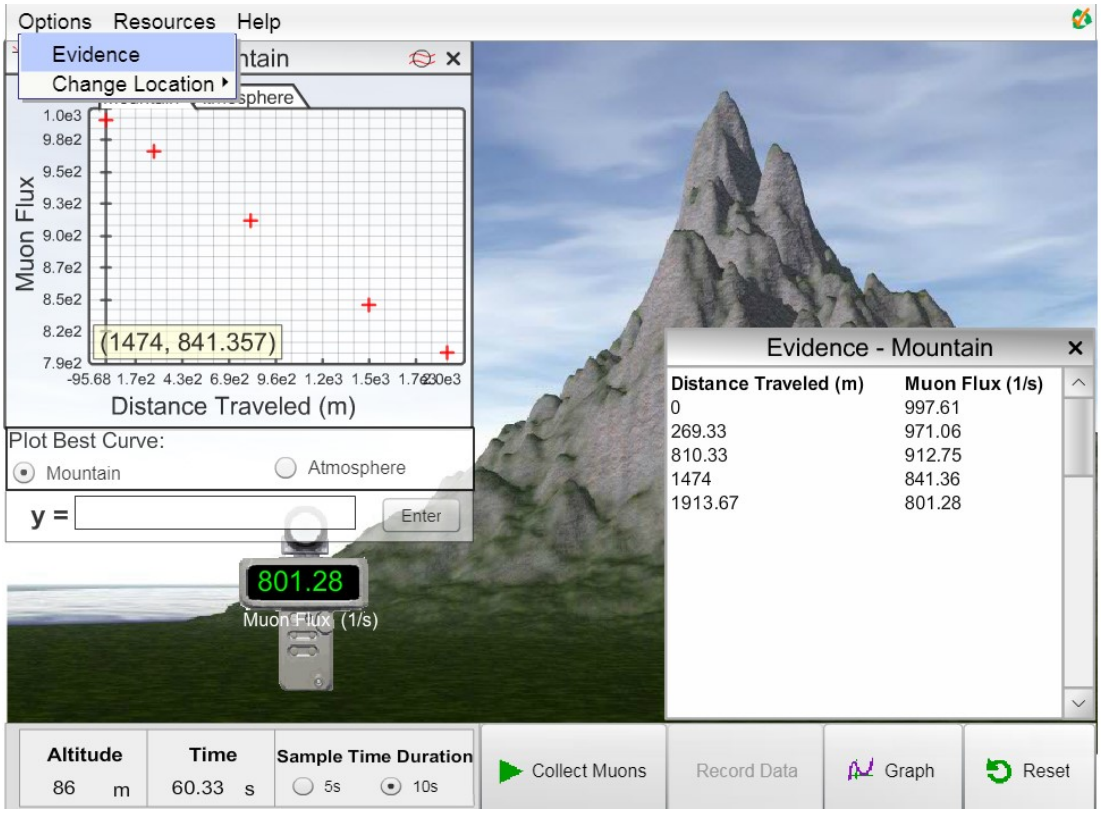
- After the data collection is finished, choose: "Record Data".

- Repeat the same procedure by decreasing the altitude by 100 m every time until you reach sea level: (Altitude = 0m).

- When data collection is complete, press: "Graph" to see your results displayed graphically.

- Then, choose: Options\ Evidence to display your measurements in table format.

- Copy the table results and paste them in an excel spreadsheet.



Picture 3.2: The graph and evidence options of the muon decay simulation

Analysis & Interpretation

- *Analysis and interpretation : Gather result from data*

  Discuss how many muons you would expect to measure at sea level, taken into account that the muons at 2km altitude are equal to the value you measured.

- Compare your prediction with the value that you measured. Discuss your results. Were they expected?

- Using the spreadsheet with the muon data, consider that the muons travel vertically downwards at velocity equal to u=0.999c. (c is the speed of light and is equal to $3*10^8$ m/s).

- Multiply the muon count rate by 10 in order to get the nr of muons collected in the 10s sampling time we used.

- Go to a new column and transform the distance travelled by the muons to time of flight. Express the muon time of flight in μs ($=10^{-6}$ sec).

- Choose the time of flight and the nr of muons columns and create a scatter plot in excel.
  Make sure that muon flux is displayed in the y-axis and time is displayed in the x-axis.

- Choose a data point, right-click it and choose: add trendline.

- Choose: the trendline to be exponential.

- Choose: Display equation on chart and then press “close”.

- Remember the equation of radioactive decay. The coefficient of time (or – “x”) in the equation appearing on your plot, is the decay constant λ. Using the value of λ you found, calculate the muon mean lifetime τ in (μs).

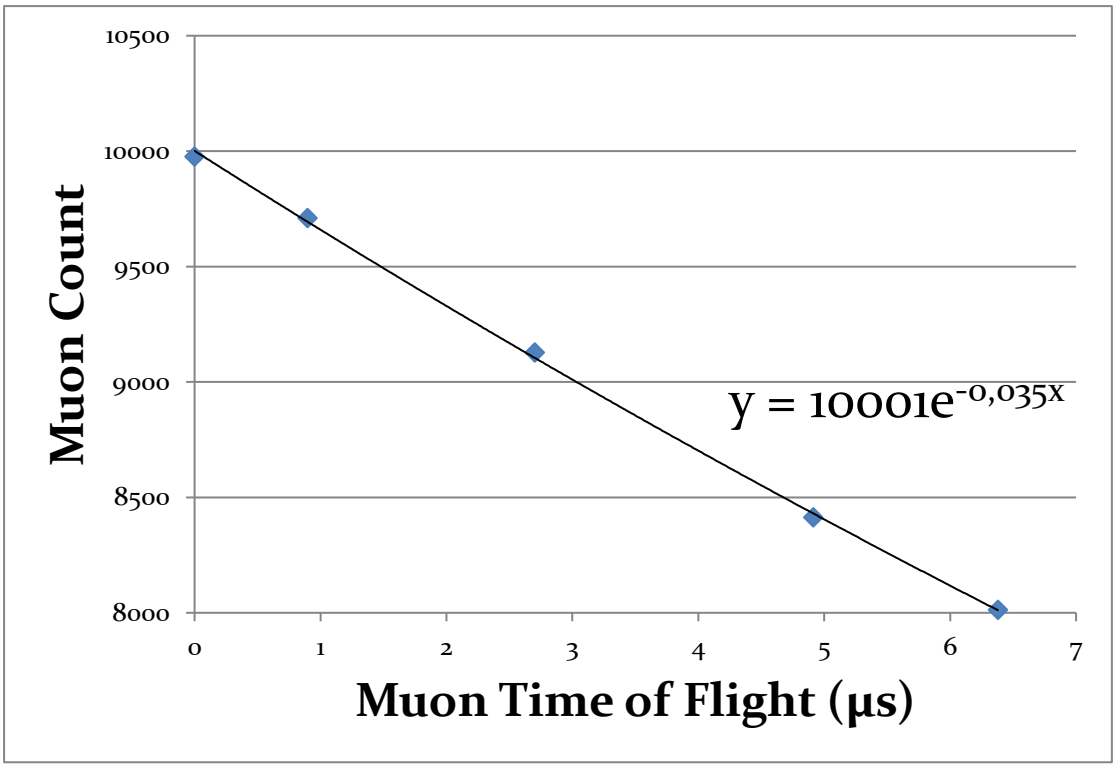
- Compare the value you calculated with the muon decay lifetime at rest from bibliography: τ= 2.2 μs. What do you observe? How can you explain it?

- Using the equation $N = No*e^{-\lambda t}$ with No being the counts you observed at 2km altitude, and $\lambda = 1/\tau$, for τ being equal to 2.2 μs.
  Calculate how many muons you would expect to reach sea level.

- We know that the muon flux at sea level is equal to 1 muon/sec.cm^2
  Assuming that you have 100% detection efficiency and that you register all downgoing muons coming from the sky: Calculate what is the size of the detector used in the simulation.

Teacher guideline

You expect the plot to be like this:



Conclusion & Evaluation

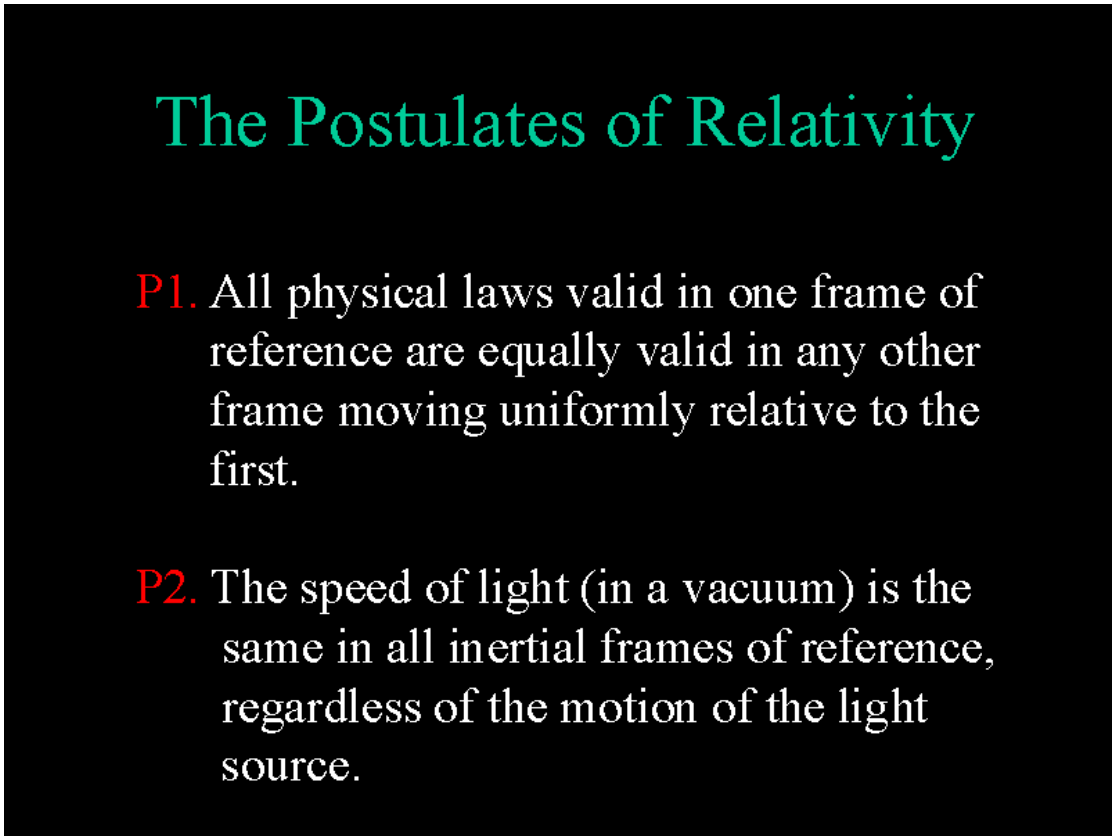
Conclude and communicate result/explanation

Do your results surprise you?
As discussed earlier, Frisch and Smith observed muon decay at rest of muons with their detector for an hour and observed: 568 counts at 2km altitude. According to the radioactive decay formula they expected 27 muons to reach sea level. Imagine their surprise when they observed not 27 but 412 muons at sea level!!

Discuss
How is this possible?
Perhaps the researchers miscalculated? The measurements were repeated and averaged. Perhaps there is some mechanism which generates more muons in lower altitudes and these muons add up? None of these happens.

To understand what happens with an analogy, imagine 568 humans who enter a spaceship immediately after their birth and go to a space voyage with speed almost equal to the speed of light. The distance they travel is 110 light years. Therefore the spaceship will take a little more than 110 years to travel to its destination.

As we know, the mean life expectancy of a human is ~70 years and the oldest person ever alive lived for 122 and a half years. How many people would you expect to be still alive by the end of the journey?
Definitely not the majority. According to the muon experiment, we would expect less than 30 people to have survived. Imagine our surprise if more than 400 of these people survives!
In other words, muons in flight seem to live longer than they live at rest!
Classical physics cannot explain this phenomenon.
This is where the special theory of relativity enters the stage!!



One of the most famous consequences of Einstein's Special Relativity Postulates is the phenomenon of Time Dilation.
According to Einstein's Special theory of Relativity, time "runs" differently for two observers with relative speed close to the speed of light.

Suppose that a person travels on a rocket with speed 98% of the speed of light. This person measures the time between two claps of his hands and finds them to be equal to 1 sec. Thus the observer at rest with respect to the clapping will measure time to=1 sec.
An observer on earth measures also the time between the two claps. Will he find 1 sec again?
The answer is no! The observer will measure approximately t = 7 sec between the two claps!!

<u>According to Einstein's Special Theory of Relativity:</u>
Assuming that two observers move with relative uniform velocity close to the speed of light:
The time interval between two events taking place at the same place with respect to an observer is called proper time and is shorter than the time measured by an observer for who the events will take place in different places.

This phenomenon is called relativistic time dilation.

The relationship relating the two time intervals is:

$$t = \frac{t_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

to: the time interval for the observer at rest with respect to the events
t : the time interval for the moving observer with respect to the events
u: the relative velocity of the two observers
c: the speed of light.

In other words, according to Einstein, time is relative and depends on the state of motion of the observer!! In other words, moving clocks tick slower than clocks at rest.

Question:
For to = 1 sec, discuss which is the minimum velocity an observer must have in order to observe a 1% difference in the time measurement.

Let's see how our new knowledge applies to the muon experiment we performed:
https://www.youtube.com/watch?v=nxQokz_qWmc

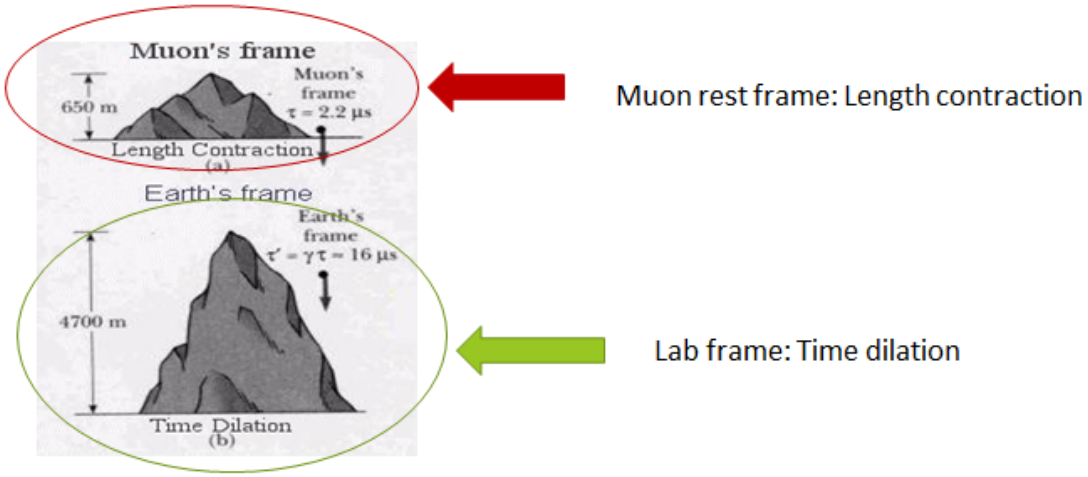
Discuss your results and explain the following:
"The muons take longer to decay in flight with respect to decaying at rest due to time dilation".

An observer sitting on the muon measures its lifetime to be equal to 2.2μs (as the muon is at rest with respect to him). The muon travels a 2km distance from the top of a mountain towards the sea level.
According to him, a 0.99c muon would have to decay after travelling 650m. However, the muon is still there after travelling 2km towards the ground. Is this a paradox?

The answer is no. As the observer on earth measures the muon to live longer than 2.2μs, the muon itself measures the elapsed distance to be shorter than 2km!
This phenomenon is length contraction.

Teacher Guidelines

For further information on time dilation and muons, check this link:
http://hyperphysics.phy-astr.gsu.edu/hbase/Relativ/muon.html#c1



Evaluation/Reflection

Create a short presentation describing your experiment, your findings and the explanation. Discuss your results with your classmates and your teacher.

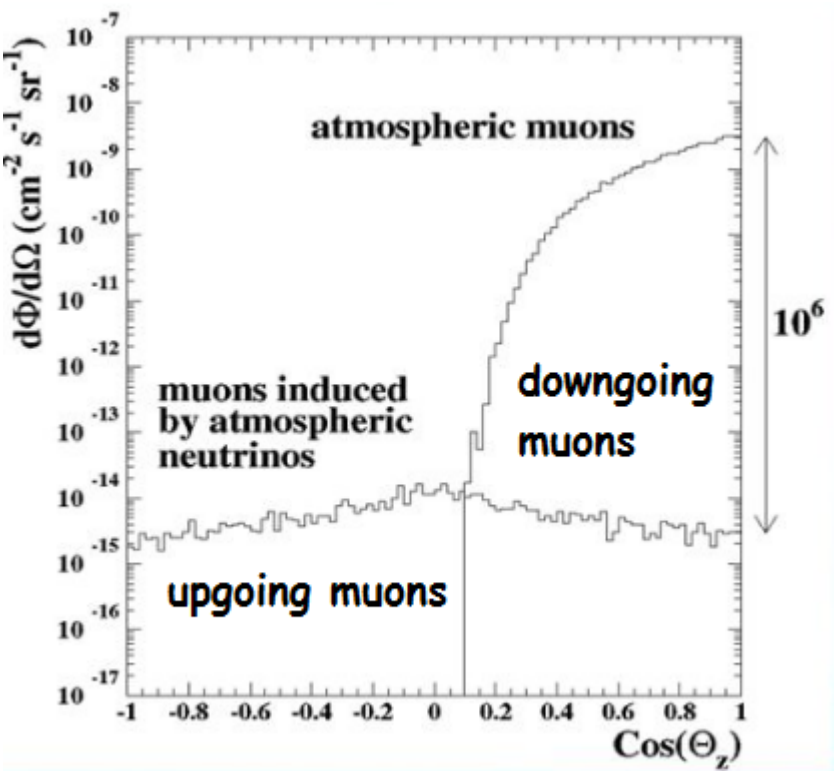
Consider other explanations

We learnt all about these mysterious cousins to the electron: the muons.
Being almost 200 times heavier than the electron and with a proper lifetime of 2,2μs, muons can be produced in elementary particle collisions both in the lab and the atmosphere:
Atmospheric muons are produced at an altitude of ~15 km in high energy collisions between cosmic ray particles and the atomic nuclei of the atmosphere's constituents.
Due to their high mass they are highly penetrative with long ranges in matter.
Using muons one can verify with simple equipment one of the most beautiful and strange theories of Physics: The Special Theory of Relativity and the phenomenon of time dilation.

As muons survive longer than anticipated, due to the relativistic time dilation, they reach the ground and the more high energetic muons can travel further below, thus creating an important background for underground arrays such as particle accelerators (the LHC for example, built 100m underground), neutrino underground and underwater experiments which search for high energy neutrinos.
Just like light pollution "blinds" our telescopes and we cannot observe clearly the night sky, the "atmospheric muon pollution" blinds the high energy neutrino telescopes. Therefore, to reduce this unwanted "background", scientists design clever triggering algorithms and take their setups to very large depths of the order of some km in water or ice.

To learn more about neutrino telescopes you can see this video:
https://www.youtube.com/watch?v=nkydJXigkRE

Teacher guideline:
You can also show your students this video on the IceCube Neutrino Telescope and investigate further at a followup activity
https://www.youtube.com/watch?v=aMnGWqoDaAA



Picture 5.2: For every interesting 'upgoing' neutrino for a neutrino telescope, 1,000,000 "unwanted" muons surviving due to time dilation are recorded by the detector!