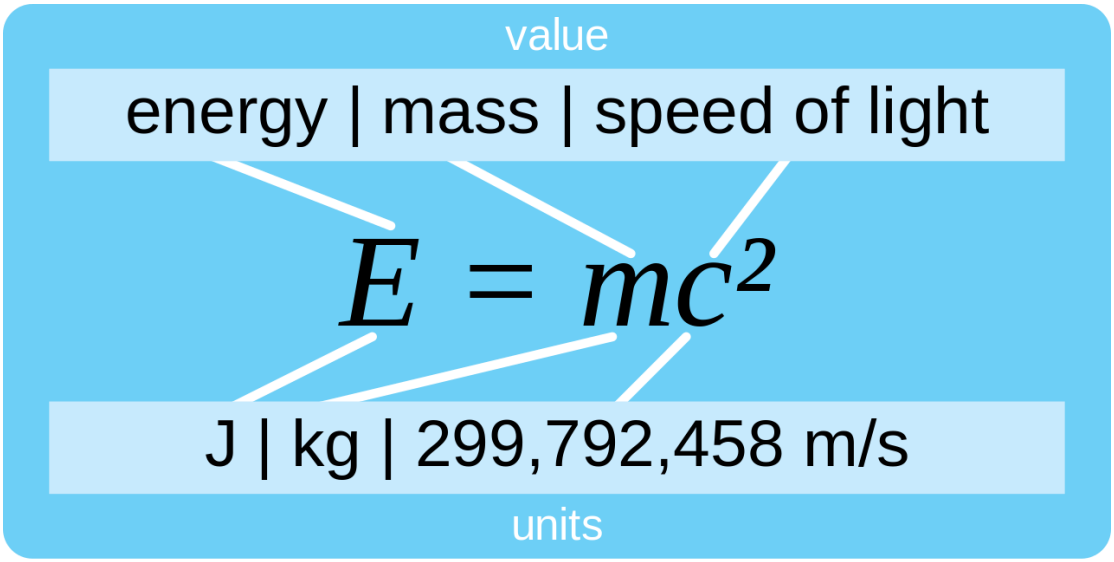
Mass – Energy Equivalence

Orientation Phase

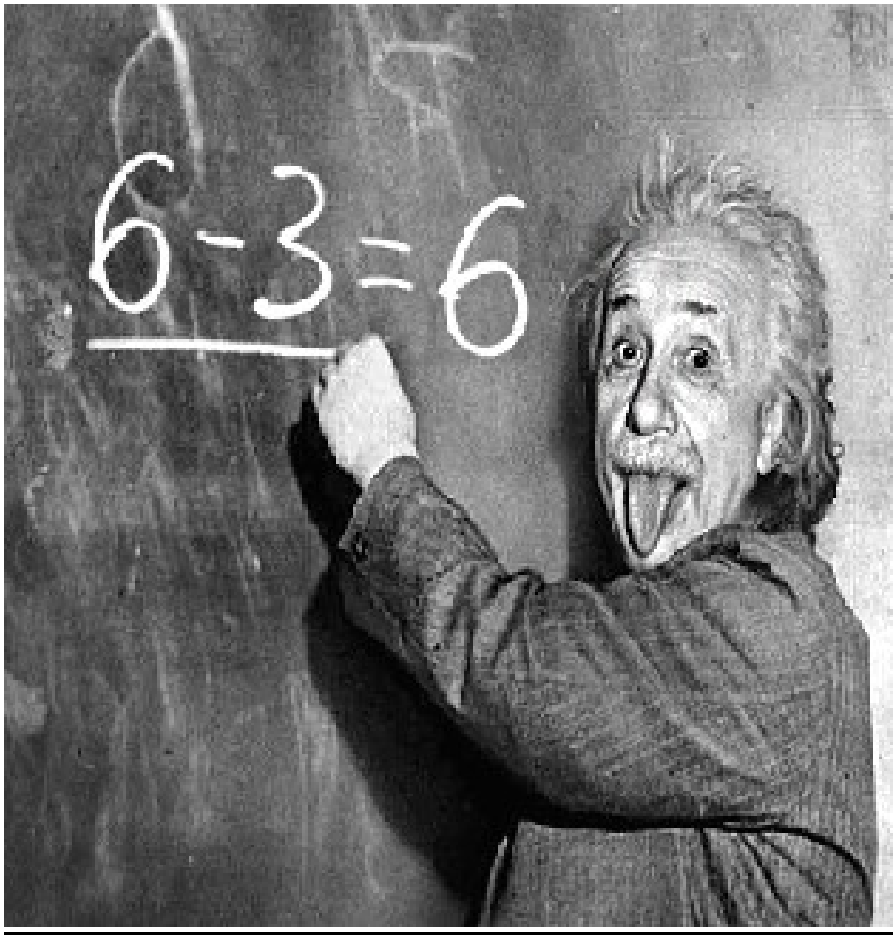
Orienting and Asking Questions

Provide Contact with the content and/or provoke curiosity



$c^2 = 89{,}875{,}517{,}873{,}681{,}800\ m^2/s^2$

Look at the picture above: You can observe the most famous equation of the 20th century. This equation was the masterpiece of Albert Einstein, the Physicist who changed forever our perception of Nature and Physics!
http://www.nobelprize.org/nobel_prizes/physics/laureates/1921/einstein-bio.html

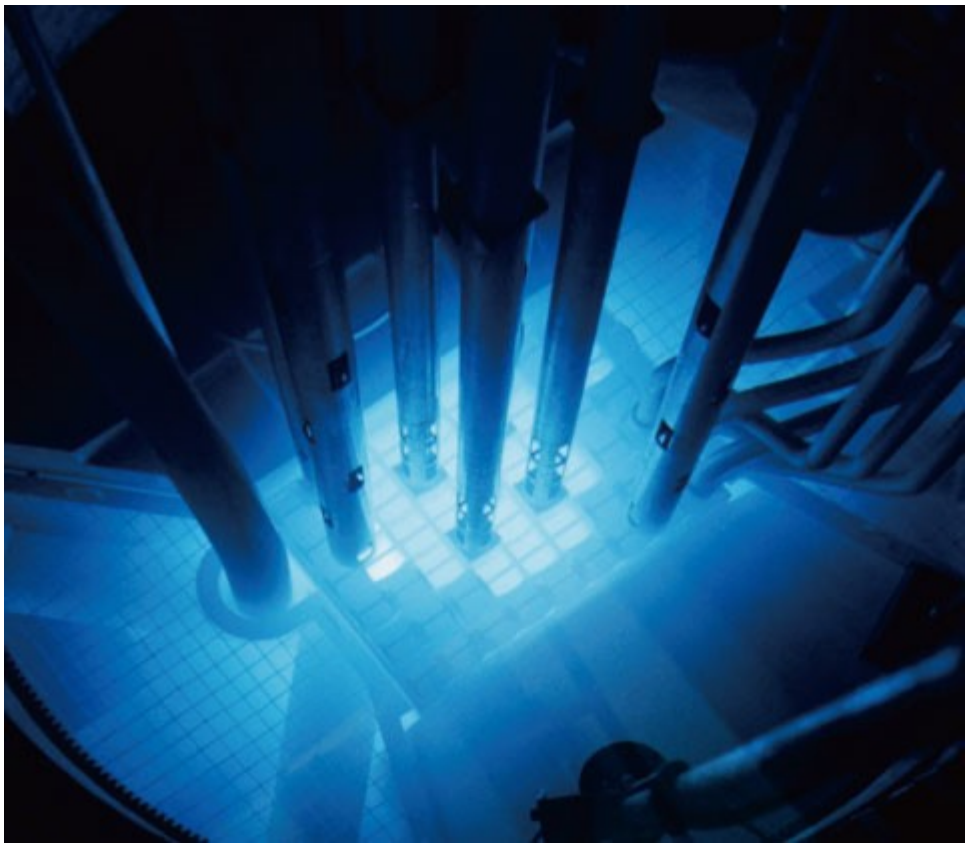


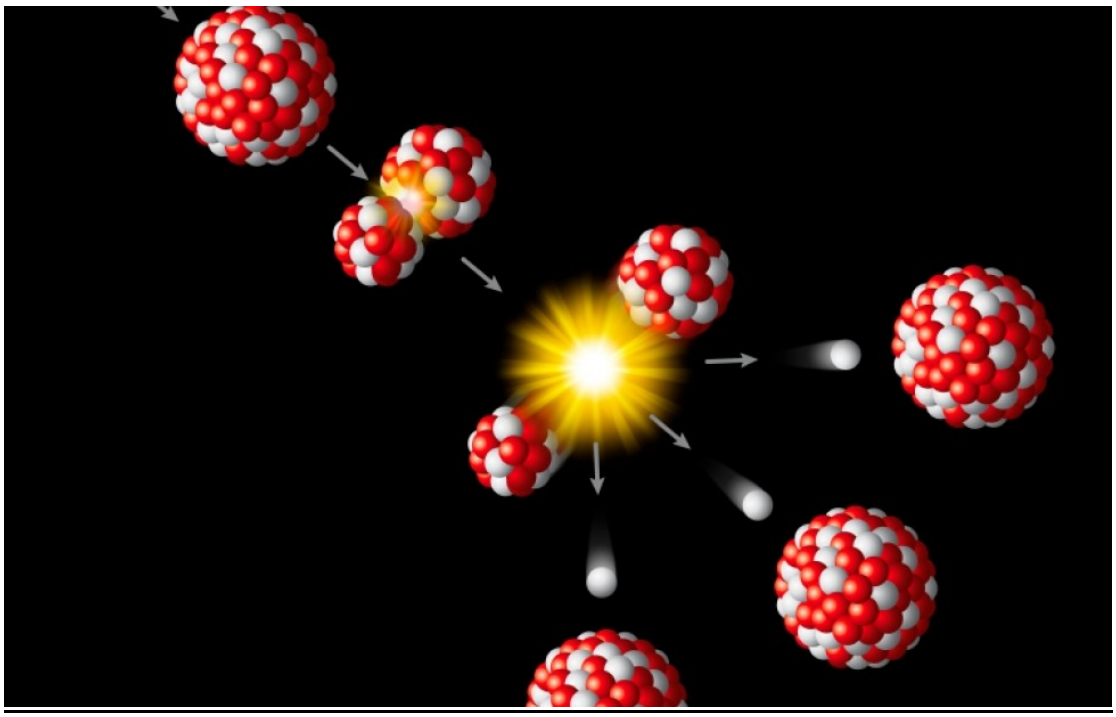
But have you ever wondered about the meaning of this equation? What about its application in science and its impact in our civilization?

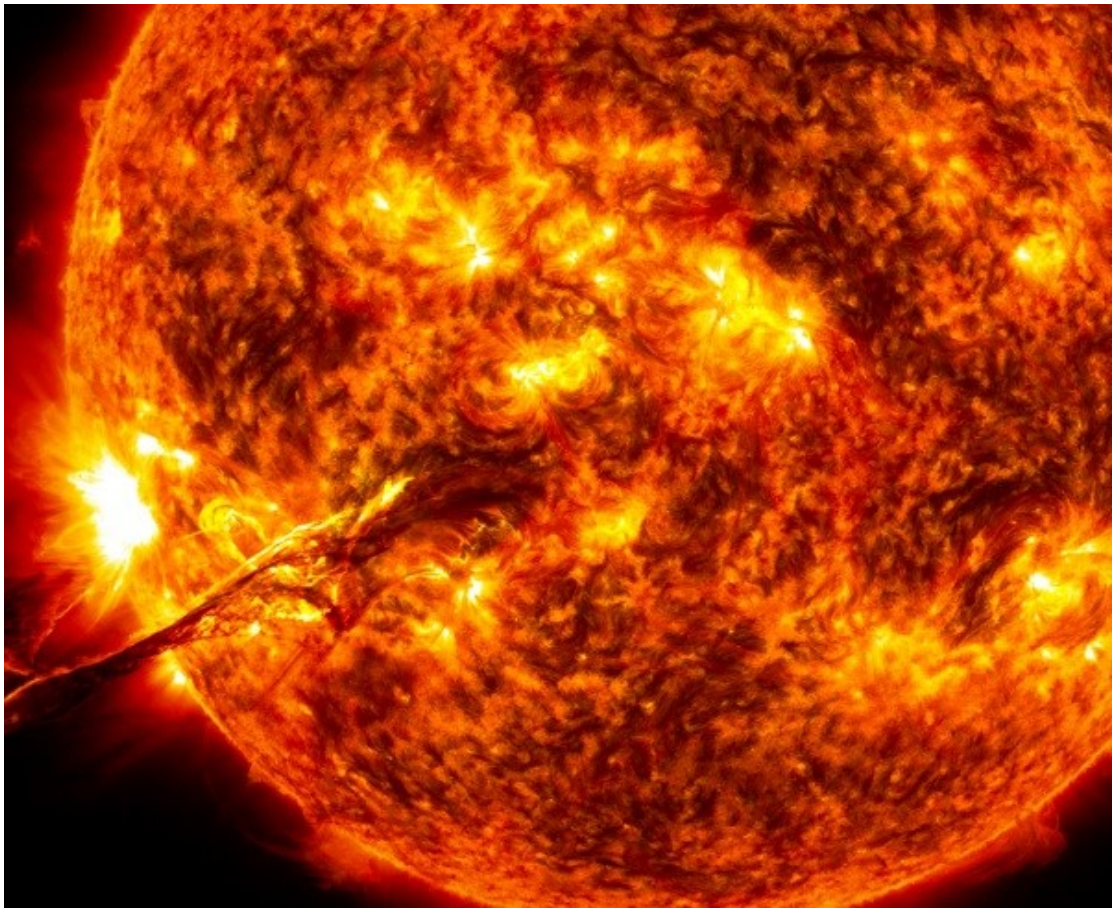
Take a look at the images below and discuss: What does each of them show us?

Try to match each picture with one of the descriptions provided:

- Nuclear Reactor
- Nuclear Bomb
- Nuclear Fission
- Energy production in the sun
- Nuclear Power Plant





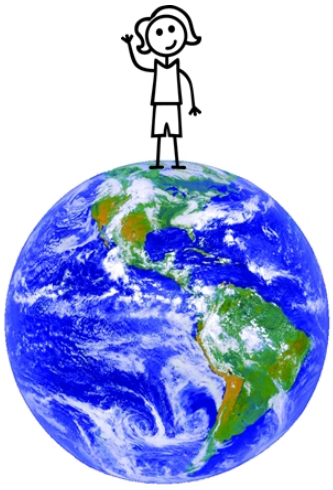


Do these images have anything in common?
Could they be connected in any way to Einstein's famous equation?

Teacher Guidelines:
Review the following concepts with your students either in this part or before starting the lesson.

Mass:
is a scalar quantity measuring the amount of matter in a body. It is measured in kg.
(Alternative definition: Mass measures the inertia of a body → meaning the resistance of a body to any change in its state of motion. More massive objects exhibit greater inertia).

Remember: Mass shouldn't be confused with Weight!



Energy :

Even though both physicists and non- physicists may use this term in an almost daily basis, there is no working definition for the term.
However, we know that energy can be transferred from a body to another, can transform from one type to another and that energy is neither created nor destroyed: It is conserved. During every physical process the total energy before and after the process is constant.
There are many forms of energy: Energy of motion (Kinetic Energy), Potential Energy, Solar energy, chemical energy, heat and others. All these though finally break up in two fundamental forms of energy: Potential and Kinetic energy.
Energy is measured in Joules (J).

Check the following videos about the conservation of energy theorem:
https://www.youtube.com/watch?v=87EoDKs5bok
https://www.youtube.com/watch?v=PplaBASQ_3M

Do you see anything in common between mass and energy??

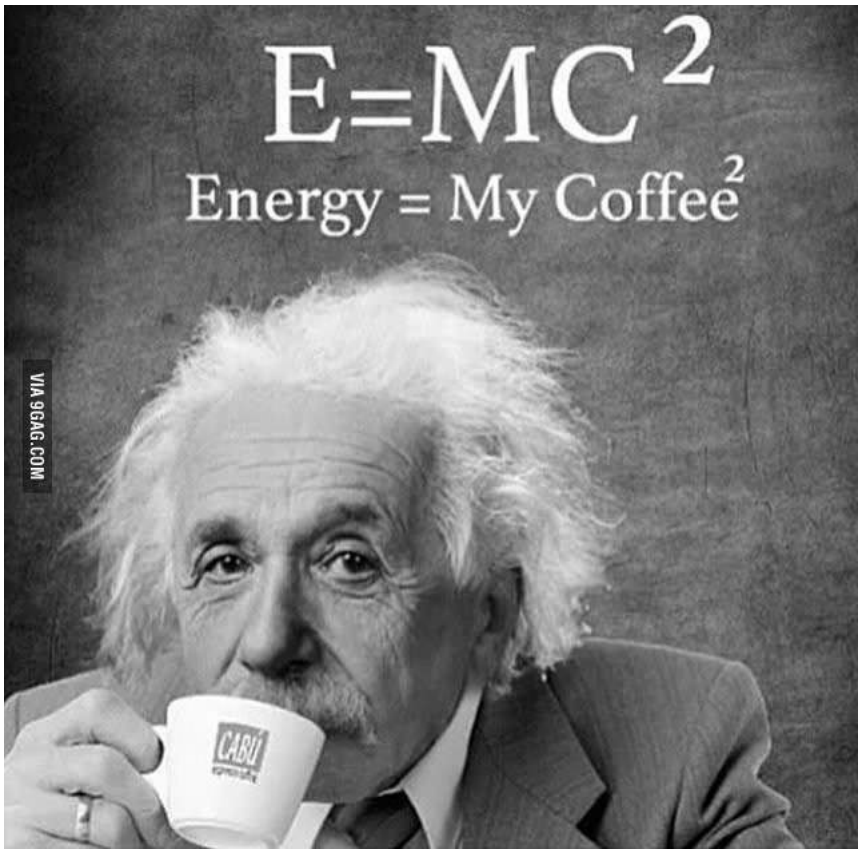
Exploratory Phase

Hypothesis Generation and Design

Define Goals and/or questions from current knowledge

Let's brace ourselves and discuss the meaning of our equation:
$E = mc^2$

Well, Einstein definitely didn't mean that:



Let's look carefully at the following video:
https://www.youtube.com/watch?v=ZKzsl1ARxeQ

As we saw:
E = Energy
M = Mass
c = speed of light in vacuum (300,000km/sec)

So Uncle Albert suggests that mass is energy and energy is mass and not only that, but the one can convert in the other also!
Wait a minute: WHAT????

We know what mass is and what energy is. And we also know that the speed of light is way too fast to perceive.
Or, do we?

So far there seems to be no proof stating that mass is a form of energy.
So far so good!

In order to check Einstein's statement, we will have to formulate an appropriate research question and investigate it:

Can mass be transformed into energy and vice versa during a physical process?

Discuss your views with your classmates.

Teacher Guidelines:

- A very nice documentary by National Geographic on the history of $E=mc^2$ https://www.youtube.com/watch?v=l__fKBSHtIo
- Minute Physics $E=mc^2$: https://www.youtube.com/watch?v=hW7DW9NIO9M
- Minute Physics $E=mc^2$ incomplete: https://www.youtube.com/watch?v=NnMIhxWRGNw
- Veritassium $E=mc^2$ and gluons: https://www.youtube.com/watch?v=Ztc6QPNUqls
- $E=mc^2$ interactive: http://labs.minutelabs.io/Mass-Energy-Scale/

Generation of Hypotheses or Preliminary Explanations

In the previous part of our lesson we posed the following question:

Can mass be transformed into energy and vice versa during a physical process?

Teacher Guidelines:
Guide the students towards the following hypothesis: A physical system emits energy. If Einstein is correct, the mass of the system must decrease.
(Likewise, when a system absorbs energy, the mass of the system must increase).

Model

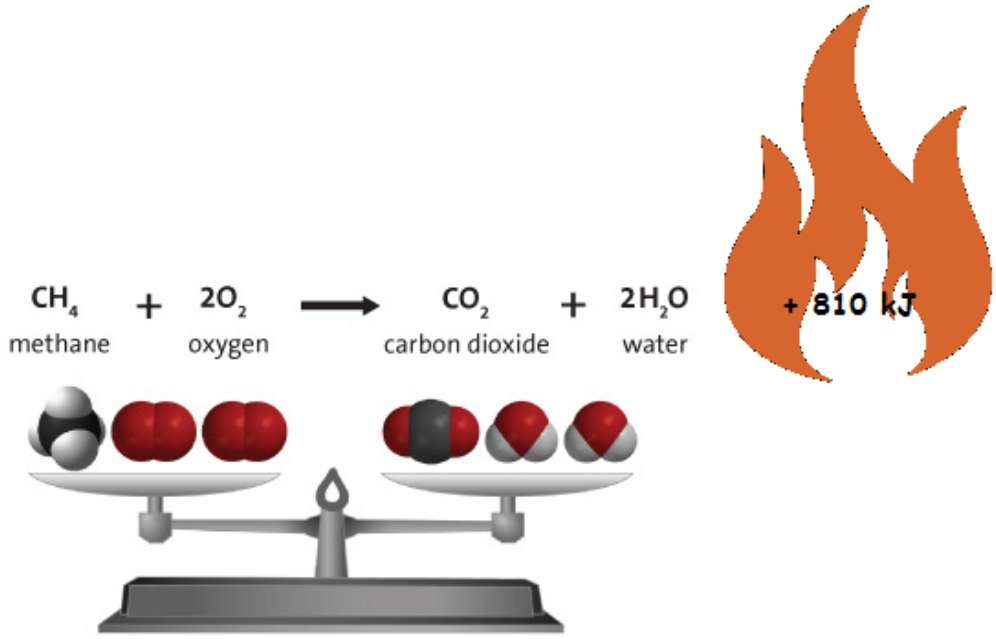
The easiest field where we can start checking our hypothesis are our well known chemical reactions:

A chemical reaction is a process that transforms one set of chemical

substances to another.

As we know from stoichiometry (for a quick reminder check here), the amount of every substance taking place in a reaction must remain the same before and after the reaction (Remember : how do we balance chemical reactions?)

Take for example the complete combustion of methane:



Count how many atoms of carbon (C), oxygen (O) and hydrogen (H) we have before and after the reaction.
The mass of the atoms above (C,H,O) is unchanged. Keeping their number constant before and after the reaction means that the total mass must be also kept constant (or else the reaction won't take place!!).
As we can see now, the combustion is an exothermal reaction. The energy released for every mole (=16g) of input methane is 810 kJ.

Short quiz:

Using: c = $3*10^8$ m/s and E = 810.000 J find the corresponding mass loss using Einstein's $E = mc^2$

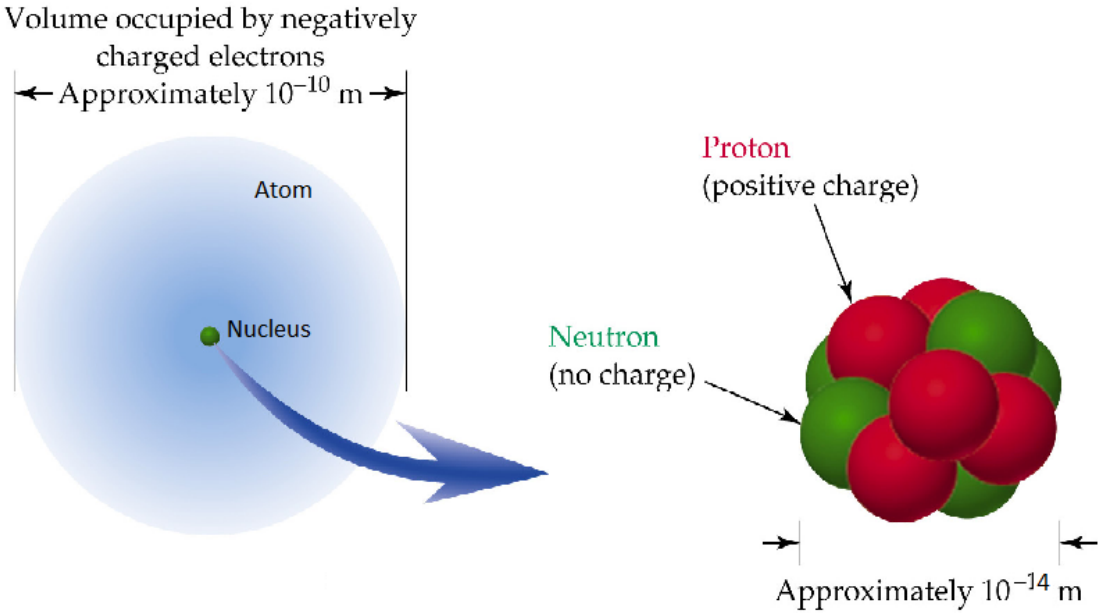
(answer: m = $9*10^{-12}$ kg)

This means that for 16 g of methane we lose 9 nanograms of mass (or for 16 tones of methane we lose 9 mg of mass) after the combustion, a difference of the order: 1/1000000000!!!

- What do you think, can we measure this quantity easily, if at all?

In order to be able to prove experimentally or disprove the Einstein equation though, we need a more "efficient" process. To do so we must probe the realm of Nuclear Physics. Why ? Because the atomic nucleus needs more energy to be kept together than an atom or a molecule. As a result, breaking apart a nucleus will result in a higher energy output.

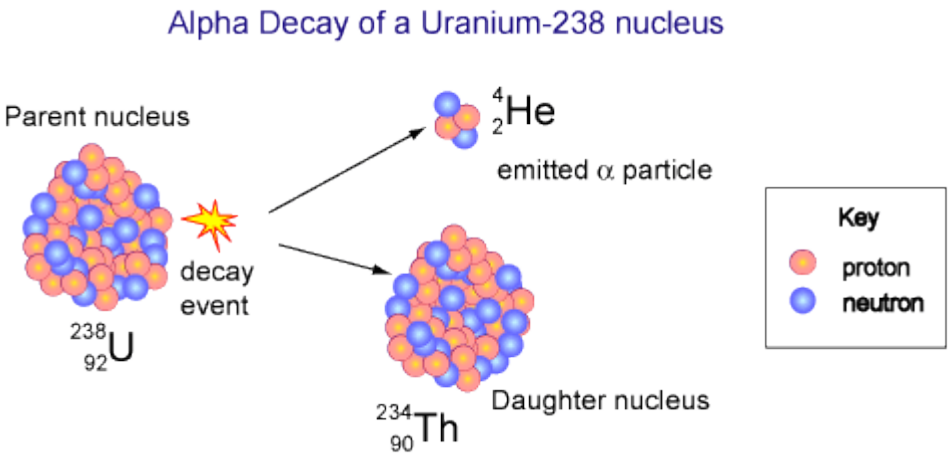
The atomic nucleus consists of protons and neutrons and is the core of every atom. Its size is of the order 10.000 smaller than the size of the atom:



During the end of the 19th and the beginning of the 20th century, scientists (Joliot, Curie, Becquerel, Rutherford) measured that some

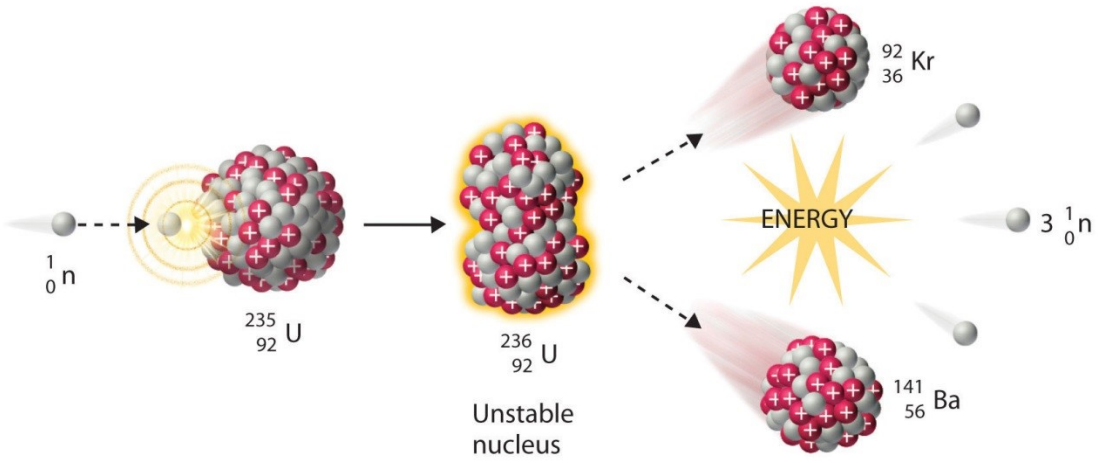
nuclei are actually unstable!
Some nuclei can “break apart” spontaneously in smaller nuclei and emit other particles and radiation (such as alpha, beta and gamma particles) in the process. Scientists called this phenomenon **radioactivity.**

Below you can see the example of an alpha decay:



Other nuclei are stable, but scientists following the pioneering work of Enrico Fermi managed to make them radioactive by bombarding them with neutrons (or protons – which one would you choose and why?-)! This is called **artificially induced radioactivity**.

If a *heavy* nucleus is bombarded by a neutron, then it can break in two almost equal in mass smaller nuclei and emit other particles *such as neutrons* in the process.
This reaction is called **nuclear fission.**



Now, after a tour in the microcosm of the nucleus we are going to investigate our hypothesis on Einstein's equation using the tool of radioactive decay provided to us by nuclear physics.

If we perform the same calculations as before, we will see that the energy emitted in a nuclear decay compared to the parent nucleus mass is 100,000 times higher compared to the chemical reaction case.
Such differences are measurable, so we will focus on the nuclear physics domain now.
Therefore, if $E=mc^2$ is accurate, we will be able to measure it in nuclear phenomena.

Planning and Experimenting

Plan Investigation

Useful Information

Units of mass and energy in the microcosm

Unit of energy

When we investigate phenomena in the nuclear domain, the units of energy we used in the macrocosm stop being relevant.
The unit of energy here is the "eV".
1 eV equals to the energy obtained by a particle with charge equal to e (electron/proton charge), which has been accelerated by a

potential difference of 1 Volt.

$$1\ eV = 1.602 \times 10^{-19} J$$

1 keV equals to 1000 eV, 1 MeV to 1,000,000 eV, 1 GeV to 1,000,000,000 eV and so on.
Practice a little bit with unit conversion. Use the following link in case you need help :
http://www.rapidtables.com/convert/energy/joule-to-ev.htm

Unit of mass

As a unit of mass, we use the atomic mass unit (1 a.m.u or 1 u).
1 u equals to 1/12 of the mass of the ^{12}C atom. That is approximately equal to the mass of the proton and the neutron.

$$1\ u = 1.6605 \times 10^{-27}\ kg$$

At the link below you will be able to find a chart of nuclear masses, measured in atomic mass units. You can click on the element of your interest and find its mass for all isotopes:
Chart

Perform Investigation

In this step you will investigate the conservation of mass, a concept known to be valid in chemical reactions, in a nuclear reaction.

$$\mathbf{n} + {}^{235}_{92}\mathbf{U} \rightarrow {}^{146}_{56}\mathbf{Ba} + {}^{88}_{36}\mathbf{Kr} + \mathbf{2n} + 178\ \text{MeV}$$

The above is a nuclear fission reaction (for more information you could exercise with this simulation).

1. Use the Chart provided above to find the mass (chart provides masses in μu) of each nuclide taking part in the reaction and write it on your notebook. We give you the neutron mass to be: 1.00866491588 u.

Nuclide	Mass (u)
n	1.00866491588
U-235	
Ba-146	
Kr -88	

2. Compare the sums of masses before and after the reaction. If they are different, calculate this difference: (Sum of masses before reaction – Sum of masses after reaction) and name it Δm_nuclear.

Sum of masses before reaction (u)	Sum of masses after reaction(u)	Δm_nuclear (u)

3. ***Is mass conserved in nuclear reactions?***

Analysis and Explanation

Analysis: Analyze evidence to discover a possible answer

Mass isn't conserved in nuclear reactions. It is now time to discuss about energy.
Examine the Δm_nuclear you found during your investigation.
Now, look again at the nuclear reaction.

$$\mathbf{n} + {}^{235}_{92}\mathbf{U} \rightarrow {}^{146}_{56}\mathbf{Ba} + {}^{88}_{36}\mathbf{Kr} + \mathbf{2n} + 178\ \text{MeV}$$

Experimental measurements provide us with the value of the energy emitted during the reaction: **~178MeV** (M stands for mega, or else 1 MeV = 1,000,000 eV). This energy is shared among the daughter nuclei produced by the fission and the neutrons in the form of kinetic energy.

Teacher Guidelines: To verify this number you can use the nuclear Q-value calculator: https://www.nndc.bnl.gov/qcalc/ . You could encourage students to browse different reactions and repeat their calculations.

For the reaction to take place, the neutron that reacts with the 235U nucleus is thermal (or else, its energy is of the order **0.025 eV**) while the U nucleus is **at rest**.

1. Apply the conservation of energy theorem before and after the reaction and write down your results. Remember that colliding nuclei here have no potential energy.

Does energy seem to be conserved?

2. Now, convert the Δm_nuclear you found from u to kg https://www.unitconverters.net/weight-and-mass/atomic-mass-unit-to-kilogram.htm.

Δm_nuclear = _____ (kg)

3. Use Einstein's equation, $E = mc^2$, to find ΔE in Joules:
here, m = Δm_nuclear (kg) and c = $3*10^8$ m/s
ΔE = _____ (J)

4. Transform the value of energy you found from Joules to MeV using this calculator
http://www.rapidtables.com/convert/energy/joule-to-ev.htm
and write down the energy emission ΔE which corresponds to the mass decrease Δm_nuclear.

ΔE = _____ (MeV)

5. Compare your result with the 178 MeV energy release provided to us by experiment.

6. To what extent does your result agree with the experiment? Discuss your observations.

Teacher Guidelines:
For this exercise, we propose that students keep an uncertainty of +/- 1 MeV in their calculations.

We propose that students use the tables for Joule to MeV conversion and for u to kg conversion if they are not familiar with these unit conversions. For the time being we need them to focus on the conclusions drawn from this experiment and not on the technical issues. However, a new iteration of the same scenario could have students practice with these conversions.

Explanation: Formulate an explanation based on the analysis of your evidence

How do you explain your results?

Was your initial hypothesis correct?
“A physical system emits energy. If Einstein is correct, the mass of the system must decrease.
(Likewise, when a system absorbs energy, the mass of the system must increase).”

Do you agree with the following statement?

“Energy and mass aren’t conserved individually during nuclear reactions. However, if we use E=mc2 then mass-energy is conserved”.

Following our results, the new equation for conservation of mass-energy is:

$$(\boldsymbol{K_{total} + m_{total}c^2})_{\boldsymbol{before}} = (\boldsymbol{K_{total} + m_{total}c^2})_{\boldsymbol{after}}$$

In which K is the kinetic energy of the particles in question.

This implies that mass times the square of the speed of light is a form of energy itself! Mass and energy aren’t conserved separately in nature

Teacher Guidelines: You can explore further with your students introducing them to the first artificial nuclear transformation taking place in 1932 by Cockroft and Walton
https://hst-archive.web.cern.ch/archiv/HST2008/bubble_chambers/Energy%20Conservation/A%20practical%20introduction%20to%20e=mc2.pdf

Consolidation Phase

Connect

Connect explanation with current scientific knowledge

According to Einstein's mass – energy equivalence:
Anything having mass has an equivalent amount of energy and vice versa, with these fundamental quantities directly relating to one another by Albert Einstein's famous formula: $\boldsymbol{E = mc^2}$

Let us watch this video of Albert Einstein explaining the mass energy equivalence:
https://www.youtube.com/watch?v=mcRmor9wT4c&feature=youtu.be&fbclid=IwAR1Z1alpEnH1pNow6qJETsyY2p5xe_dfyV3Q0GTYUQMovicARaokH7AIYQ0

You can browse this application to see the amount of energy that corresponds to the masses of objects we know from our everyday

life.
http://labs.minutelabs.io/Mass-Energy-Scale/

Implications of the mass-energy equivalence:

When mass transforms into energy

When a system undergoes a process which results in the emission (or absorption) of energy equal to E, then the system's mass will decrease (or increase) by :

$$\mathbf{\Delta M = \frac{E}{c^2}}$$

This effect is exploited in nuclear reactors which harness the energy emitted in the process nuclear fission. As we saw in our investigation, during the fission process a part of the parent nucleus mass is transformed into kinetic energy of the product particles. Neutrons produced in a nuclear fission then trigger the fission of other nuclei, which in turn produce more neutrons continuing with what is known as a 'chain reaction'.

For an overview of mass energy equivalence and the fission process you can look at the following video:

https://www.youtube.com/watch?v=9gERUtbtkRc

When energy transforms into mass

We saw that mass can transform into energy. However, Einstein's famous equation works both ways.

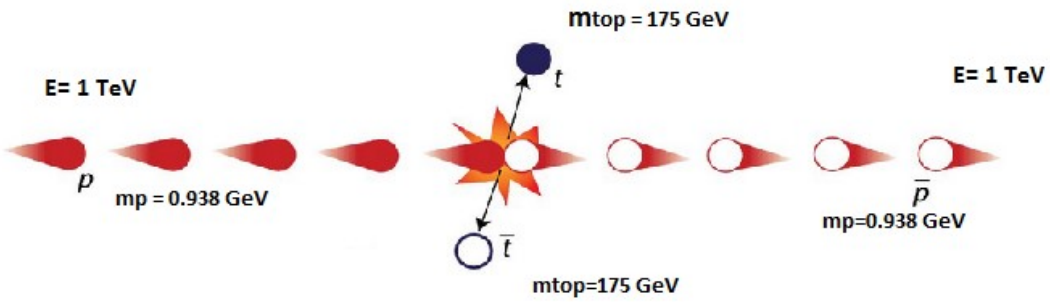
$$\boldsymbol{E = mc^2}$$

implies that energy can also be transformed into mass. If the

amount of energy produced during an interaction, which obeys several conservation laws, is equal to or higher than the mass times c^2 of an elementary particle (or more), then this elementary particle will be produced while it did not exist before.

This effect is exploited in particle accelerators, machines in which scientists accelerate particles such as protons, electrons or heavy ions and collide them with other particles. The energy produced during these collisions can be transformed into mass and thus result in the production of new, even heavier, particles!

In the following figure you observe the collision of a proton and an antiproton (proton's anti-particle) with kinetic energies of the order of 1 TeV and masses of the order of 1 GeV (1 thousandth of a TeV). This collision results in the production of top and anti top quark among other products.



Observe that the top (and anti-top) quark has mass equal to approximately 175 GeV which is 175 times higher than the mass of the proton!

https://www.youtube.com/watch?v=_wxFYcEuLYo

Let us watch these videos about mass and energy equivalence which summarizes our discussion so far:
https://www.youtube.com/watch?v=6569GBcQKTs
https://www.youtube.com/watch?v=X0232kyTsO0&t=5s

Can you think of any possible applications of Einstein's mass-energy equivalence in real life?

Teacher Guidelines:

This question can serve as a starting point to primarily discuss both nuclear energy and nuclear weapons.

The following video introduces the concept of chain reaction in a funny and engaging fashion:

https://www.youtube.com/watch?v=0v8i4v1mieU

Now that we have understood Einstein's equation and its application in nuclear fission, we can understand how nuclear reactors and nuclear bombs function:

Nuclear reactors:
(pedagogical intro)
https://www.youtube.com/watch?v=MGj_aJz7cTs
(scientific intro)
https://www.youtube.com/watch?v=1U6Nzcv9Vws
(a tour in MIT's research reactor)
https://www.youtube.com/watch?v=5QcN3KDexcU

Nuclear Bombs

(HOW IT WORKS: The Atomic Bomb – from 11.00 and on)
https://www.youtube.com/watch?v=-NcowCrkkoo

(The scale of nuclear bombs)
https://www.youtube.com/watch?v=fs1CIrwg5zU

(Dropping the bomb: a BBC short documentary about the last moments before the atomic bomb was dropped in Hiroshima in 1945)
https://www.youtube.com/watch?v=3wxWNAM8Cso

(A crash course in the history of atomic bomb from the early 40's up to today)
https://www.youtube.com/watch?v=w4q1fG1vh5I